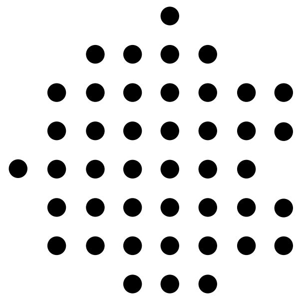
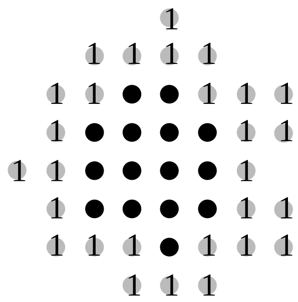
Information Statistics II

Lecture 6. Skeleton and mathematical morphology

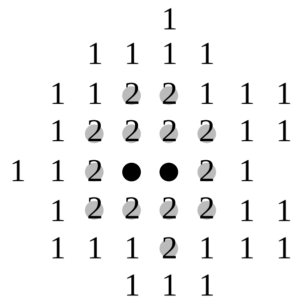
Distance transformation



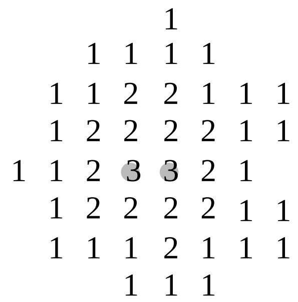
1. Original image



2. "1's" are assigned to pixels which are on the 8-neighborhood boundary.



3. "2's" are assigned to pixels which are connected to the pixels with "1."

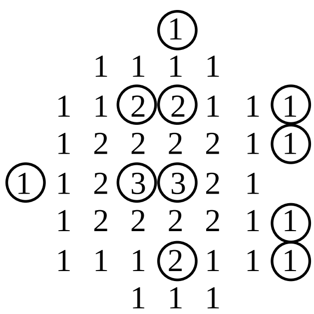


4. "*n*'s" are assigned to pixels which are connected to the pixels with "*n*–1."

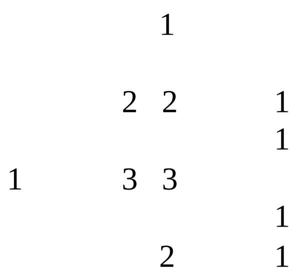
Fig. 1. Distance transformation.

The pixel values of distance-transformed image indicates the distance from the boundary to the pixels.

Skelton and distance transformation



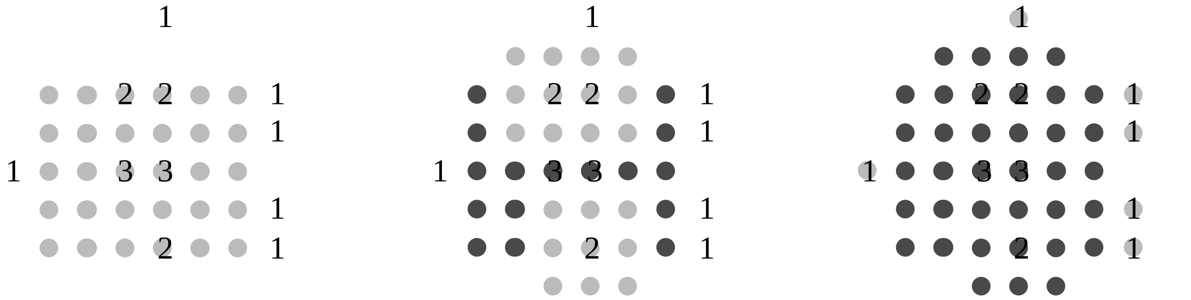
◯ pixels whose values are greater than or equal to every pixel in their 8-neighborhood.
(local maxima of pixel values)



skeleton = set of maxima.

Fig. 2. Creating skelton from a distance transformed image.

Restoration of image from skelton



1. squares of "size 2" are centered on the pixels with "3."

2. squares of "size 1" are centered on the pixels with "2."

3. one-pixel points are centered on the pixels with "1."

Fig. 3. Restration of the original image from a skelton.
The distance from the center of the restoring square of size *n* is *n* – 1.

Skeleton and mathematical morphology

nB	"circle" of radius *n* *NB. B* can be a structuring element of arbitrary shape.
$X \ominus nB^S$	Parts of the distance-transformed image where the pixel values are greater than *n*+1
$S_n(X) = (X \ominus nB^S) - (X \ominus nB^S)_B$	Set of pixels that belong to the skeleton and whose value is *n*+1
$SK(X) = \bigcup_n S_n(X)$	Skeleton
$X = \bigcup_n \left[S_n(X) \oplus nB \right]$	Restration of the original image from the skeleton

Proof)

$$\left[S_n(X) \oplus nB\right] = \left[(X \ominus nB^S) - (X \ominus nB^S)_B\right] \oplus nB$$

$$= (X \ominus nB^S) \oplus nB - (X \ominus nB^S)_B \oplus nB$$

$$= (X \ominus nB^S) \oplus nB - (X \ominus nB^S \ominus B^S \oplus B) \oplus nB$$

$$= X \ominus nB^S \oplus nB - X \ominus (n+1)B^S \oplus (n+1)B$$

$$= X_{nB} - X_{(n+1)B}$$

$$\bigcup_n \left[S_n(X) \oplus nB \right] = \bigcup_n \left[X_{nB} - X_{(n+1)B} \right]$$

$$= (X - X_B) \cup (X_B - X_{2B}) \cup (X_{2B} - X_{3B}) \cup \cdots$$

Since $(A - B) \cup (B - C) = A - C$ and $X_{nB} = \varnothing$ for sufficiently large *n*, we get from the above equation $\bigcup_n \left[S_n(X) \oplus nB \right] = X$.