Information Statistics II

Lecture 7. Opening transform / Color morphology and morphology on lattice

Opening transform

The opening transform assigns to each pixel the maximum size of structuring element with which the pixel is preserved by opening. The opening transform of a pixel *x* in an image *X* with a structuring element *B*, denoted $O_{X,B}(x)$, is defined as follows:

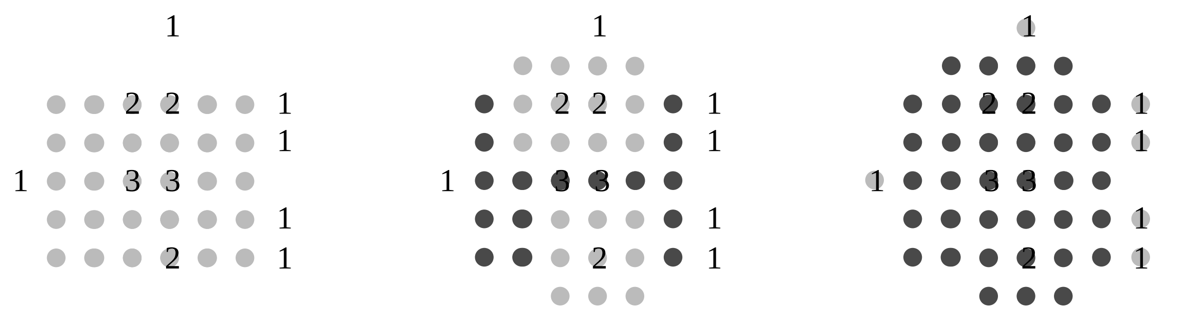
$$O_{X,\,B}(x) = \max\,\{r \mid x \in X_{rB}\}\ . \quad (1)$$

Using this expression, the pattern spectrum of image *X* with structuring element *B*, denoted $\mathrm{PS}_X(r, B)$, is expressed as follows:

$$\mathrm{PS}_X(r, B) = A(\{x \mid O_{X,\,B}(x) = r\}) \quad (2)$$

where *A* denotes the cardinality of a set, i. e. area of the set in an image, or number of pixels of the set in the discrete case.

Opening transform is closely related to distance transformation and skelton. Figure 1 is the restoration process of image from its skelton, the same as Fig. 3 in Lecture 6. This skelton is derived using 8-neighborhood, equivalent to the square structuring element of 3x3 pixels. At the step 1, squares of size 2 are centered on the pixel with distance 3. The pixels which are included in the squares of the size 2 homothetic expansion of the square structuring element of 3x3 pixels are not removed by opening but removed by opening of size 3, so that they are assigned opening transform value 2. At the step 2, squares of size 1 are centered on the pixels with distance 2 and overlapped. The pixels included in the sqaures of size 1 but not included in the previously placed squares of size 2 are not removed by opening of size 1 but removed by opening of size 2, so that they are assigned opening transform value 1. The opening transform value 1 is assigned similarly.



1. squares of "size 2" are centered on the pixels with "3."

2. squares of "size 1" are centered on the pixels with "2."

3. one-pixel points are centered on the pixels with "1."

Fig. 1. Steps of the restoration process from the skelton.

Color morphology

Each pixel value of color images is a vector of several color components, for example R, G, B. Definition of morphological operations for color images requires definitions of "maximum" and "minimum" operations of vectors.

One straightforward solution is applying ordinary morphological operations to each component independently. For example, an RGB image is processed by applying morphological operations to R-, G-, and B- image separately with the same structuring element. However, by this method, it is possible that R- and G- components of a portion of an object is removed but B-component is preserved. In this case, this portion is preserved but its color is completely modified (Fig. 2).

A more sophisticated way of definition of color morphological operation is achieved by introducing ordering of vectors. Morphological operation is ultimately reduced to maximum and minimum operations of pixel values. Thus it is possible to define morphological operations if "maximum" and "minimum" is defined for every subset of the considered set of vectors. This kind of the system of set of vectors and operations is called "lattice."

Definitions

A relationship "≤" is *ordering* or *partial ordering* in a set X if

i) for all $x \in X$, $x \leq x$ (reflexivity)
ii) for all $x, y \in X$, $x = y$ if $x \leq y$ and $y \leq x$ (anti-symmetry)
iii) for all $x, y, z \in X$, $x \leq z$ if $x \leq y$ and $y \leq z$ (transitivity)

A set X is *semiordered set* (*partially ordered set* or *poset*) if an ordering is defined for some pairs of elements of X. We call that x is "smaller than or equal to" y and y is "greater than or equal to" x with respect to the ordering ≤ if $x \leq y$.

A set X is *totally ordered set* if an ordering is defined for *any* pair of elements of X. All the elements of a totally ordered set form a linear sequence of elements ordered by this ordering. For example, a subset of integer (e.g. gray scale pixel value) is a totally ordered set with respect to the ordering "≤ (in usual sense)".

An element a is a *maximal* of a subset A of X if no element of A is "greater than or equal to" a, and a *minimal* if no element is "smaller than or equal to" a. An element a is the *maximum* of A if a is "greater than or equal to" *all* the elements of A, and the *minimum* of A if a is "smaller than or equal to" all elements of A.

The set of all the elements of X "greater than or equal to" ("smaller than or equal to") all the elements of the subset A of X is the *upper* (*lower*) *bound* of A. The minimum (maximum) of the upper (lower) bound is *supremum* (*infimum*) of A. Note that the supermum or infimum are *not* necessary to be an element of A: if the supremum (infimum) is an element of A, it is the same as the maximum (minimum) of A.

If there exist the supremum and infimum for all pairs of elements of X, the argebraic system of set X and operations that calculates the supremum and infimum is called a *lattice*.

Examples

Figures in Fig. 3 are called Hasse diagrams, which indicate orderings between elements of a set. (a) is a totally ordered set. In (b), the supremum of c and e is c: the supremum of c and d is b, neither c or d. (c) is an ordering among vertices of a cube.

Color morphology and lattice

From the above discussion, we get that morphological operations can be defined if ordering of vectors that indicate color pixel values is defined to form a lattice. There are several coordinate systems of vectors to express colors, for example RGB, YIQ, etc. The most usual ordering is comparison of a linear combination of components. However, there are many more possible orderings and investigating practically useful ordering is under study.

Moreover, there is another theory of morphology that starts from ordering operations on a lattice (See ref. 1).

Reference

1. H. J. A. M. Heijmans, *Morphological Image Operators*, Academic Press (1994).

2. 小倉久和, *情報の基礎離散数学*, 近代科学社 (1999).

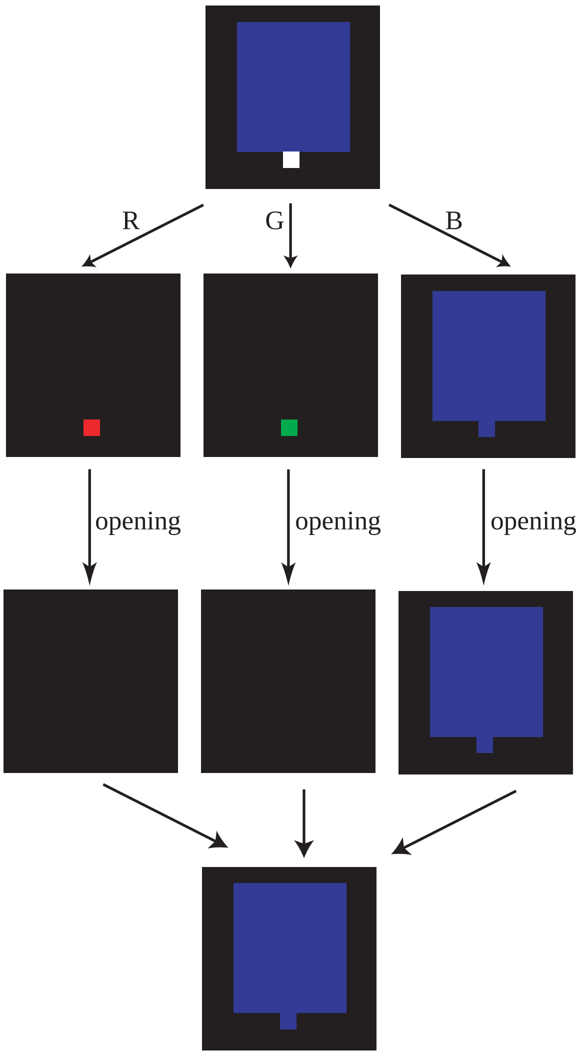


Fig. 2. Example of component-wise operation on a color image.

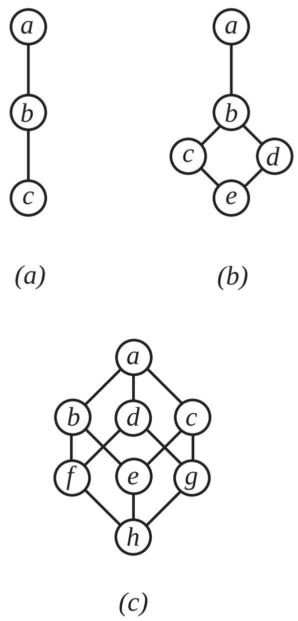


Fig. 3. Examples of lattice in Hasse diagrams.