TEXTURE PRIMITIVE DESCRIPTION USING SKELETON

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Abstract A novel method of texture primitive description using morphological skeleton is proposed. The skeleton of an object has the property that it is reduced to one point when the structuring element used for the skeletonization is exactly homothetic to the object. This method applies this property. This method minimizes the number of pixels contained in the skeleton. If we assume that the texture is composed of one primitive, the structuring element minimizing the number of pixels is homothetic to the primitive of the texture because of the above property of the skeleton. Simulated annealing is employed for the minimization. This method has an advantage that it requires no assumption on the sizing distribution of grains in the texture.

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1. Introduction

Texture recognition and discrimination are important aims of image processing, as well as object shape recognition in images. A lot of texture analyzing methods have been proposed, and the texture classification and segmentation are main objectives among them. The texture classification and segmentation requires characterization of textures, i.e. evaluation of features describing local or global characteristics of the target texture.

According to [1], the texture characterization approaches can be divided into four categories: statistical, geometrical, model-based and signal processing. We have recently investigated several approaches that are categorized into geometrical ones [2][3]. The geometrical approach considers a texture to be composed of primitives, and attempts to describe the shapes of primitives. We applied the concept of morphological size distribution [4][5] to the primitive description. We assumed a distribution of grain sizes in a texture. For example, we assume that the target texture contains grains whose shapes are homothetic to one primitive and whose sizes are uniformly distributed. In this case, the size density function relative to such structuring element that is homothetic to the primitive will be uniform. We employed simulated annealing for finding the optimal structuring element that makes the size density function uniform. Other approaches estimating the sizing distributions of grains are found in [6] [7]. These approaches assume that the granulometric moments of the primitives are known.

We propose in this paper a novel method of the texture primitive description that requires no assumption on the distribution of grain sizes or the granulometric moments of the primitives. We employ the morphological skeleton [8] [9] for this method. The most commonly employed morphological skeleton of a binary object is explained intuitively as follows: Suppose covering the object with homothetic magnifications of a structuring element in the following way. At first we locate the largest magnification included within the object, and cover the object by sweeping the magnification within the object. Then gradually smaller magnifications are employed for covering the residual area until the whole object is covered. Figure 1 shows an example. In this case the structuring element is a circle, and Fig. 1(a) shows some of the magnified structuring elements required for covering the object by the above procedure. The skeleton is defined as the union of the origins of all the employed magnifications of the structuring element, as shown in Fig. 1(b).

The skeleton varies depending on the shape of the structuring element. If the structuring element is homothetic to the object, the object is covered with only one magnification of the structuring element. In this case the skeleton is reduced to one point. We consider here obtaining the skeleton from a binary texture. It is derived from the above property that the total number of pixels within the skeleton is the minimum when the structuring element is homothetic to the primitive, if we assume that the texture is composed of one primitive, i.e.

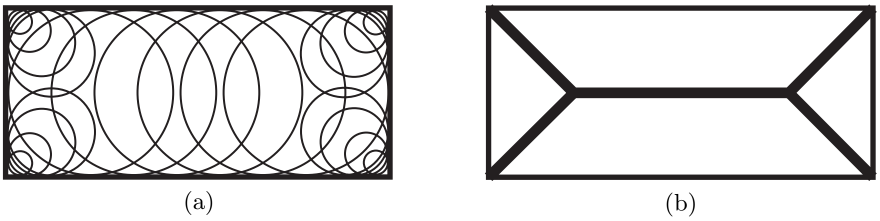
(a) (b)

Figure 1. An example of skeletonization. (a) Covering a rectangle with circles. (b) Skeleton (thick lines).

contains grains that are magnifications of the primitive. This indicates that the primitive is described by the optimal structuring element minimizing the total number of pixels within the skeleton. We employ simulated annealing for this optimization. This primitive description method has an advantage that no assumption on the sizing distribution of grains in the texture is required.

2. Method

This method obtains the primitive as the optimal structuring element when the total number of pixels in the skeleton is minimized. We explain the formal definition of the skeleton and the whole optimization procedure by simulated annealing in the following.

2.1 SKELETONIZATION

We employ the most common morphological skeleton in this method. Let X be a binary image set, and B be a structuring element. The skeleton $SK(X)$ is defined as follows:

$$SK(X,B) = \bigcup_{n=0}^{\infty} SK_n(X,B), \tag{1}$$

$$SK_n(X,B) = (X \ominus n\check{B}) - (X \ominus n\check{B})_B, \tag{2}$$

where $\ominus$ denotes the Minkowski set subtraction, $\check{B}$ denotes the symmetrical set of B with respect to the origin of B, and nB is the $n-$times homothetic magnification of B, defined as follows:

$$nB = B \oplus B \oplus \ldots \oplus B \quad ((n-1) - \text{times of } \oplus), \tag{3}$$

$$0B = \{0\}. \tag{4}$$

X_B denotes the opening of X by B, defined as follows:

$$X_B = (X \ominus \check{B}) \oplus B, \tag{5}$$

where $\oplus$ denotes the Minkowski set addition.

The grayscale image composed by assigning pixel value n to the pixels in $SK_n(X,B)$ is referred as the medial axis transform. The original binary image can be reconstructed by the medial axis transforms of all n where $SK_n(X,B) \neq \emptyset$. Our method employs the skeleton $SK(X,B)$ only.

2.2 OPTIMIZATION BY SIMULATED ANNEALING

Our method minimizes the number of pixels within the skeleton by the iterative modification of the structuring element. Even if we restrict ourselves to the consider 5×5 binary structuring elements, there are 2^{25} possible structuring elements. Thus we apply the simulated annealing to the optimization. The optimization procedure is as follows:

1 Assigning an initial structuring element to B.

2 Deriving the skeleton $SK(X,B)$ by the above procedure.

3 Assigning a modification of B to B'_i, where i is the iteration index.

4 Deriving the skeleton $SK(X,B'_i)$.

5 Evaluating $N(SK(X,B))$ and $N(SK(X,B'_i))$, which are the numbers of pixels contained in $SK(X,B)$ and $SK(X,B'_i)$, respectively.

6
  - If $N(SK(X,B)) > N(SK(X,B'_i))$, replacing B with B'_i and go back to the step 3,
  - If $N(SK(X,B)) \leq N(SK(X,B'_i))$, replacing B with B'_i by a small probability to avoid reaching a local minimum, and then go back to the Step 3.

7 Iterating Steps 3–6 until the replacement of B with B'_i does not occur anymore.

Details of the structuring element modification in Step 3 and the probability in Step 6 are explained in the following.

Modification of structuring element. The structuring element used for the skeletonization is regarded as the definition of the unit distance in the sense of the distance transformation. The distance between each pixel within the structuring element and origin is defined as unity. Thus we modify the structuring element under the condition that it contains the origin and is convex in the sense of 8-pixel neighborhood.

We assume here a binary structuring element. The structuring element is modified at an iteration in the following sub-procedure:

1 Choosing randomly one pixel in a fixed area (e.g. 5×5 pixels).

2 Altering the chosen pixel, i.e. the chosen pixel is altered to zero if it is unity, and to unity if it is zero.

3 If the altered pixel violates the above condition of convexity, this alteration is cancelled and the procedure restarts from the random choosing. Otherwise this alteration is accepted and the resultant structuring element is used as B'_i in the step 3 of the main optimization algorithm.

Probability of the replacement of B with B'_i. In Step 6, we compare $N(SK(X,B))$ and $N(SK(X,B'_i))$, and decide whether the modification of the structuring element is accepted or not. If $N(SK(X,B)) > N(SK(X,B'_i))$, the modification from B to B'_i is always accepted and B in the next iteration is replaced with the current B'_i. If $N(SK(X,B)) \leq N(SK(X,B'_i))$, the modification is accepted with the probability $P(B,B'_i)$, defined as follows:

$$P(B,B'_i) = \frac{1}{1+\exp(\frac{N(SK(X,B'_i))-N(SK(X,B))}{T(i)})}, \tag{6}$$

where $T(i)$ is called the temperature parameter at the ith iteration. $T(i)$ decreases with the progress of iteration. If the modification is not accepted, it is cancelled.

3. Experiment

We carried out an experiment using an example binary texture. Figure 2 shows the example texture. Figure 3(a)(b) and (c) show the skeletons by various structuring elements. The employed structuring element is shown beside each skeleton image. One dot corresponds to one pixel and the symbol "+" indicates the origin in the illustrations of structuring elements. The number shown beside each skeleton is the number of pixels in each skeleton. This result shows that the number of pixels is small when the structuring element resembles the primitive. We optimized the structuring element within 5 × 5-pixel square by the algorithm shown above. We defined the temperature parameter $T(i)$ in this case as follows:

$$T(i) = 100 \times 0.98^{(i-1)}. \tag{7}$$

and iterated the procedure 100 times. We tried the optimization several times, and chose the best structuring element shown in Fig. 3(d). The number of pixels contained in the skeleton by this structuring element is smaller than those in Fig. 3(a)(b) and (c). It follows that the structuring element (d) resembles the primitive of this texture better than (a)(b) and (c) in the sense of the similarity of the structuring element and the primitive.

Figure 2. An example texture.

Figures 4 and 5 show another experimental result. Figure 4 is extracted from Brodatz texture database [10] and binarized. The optimal structuring

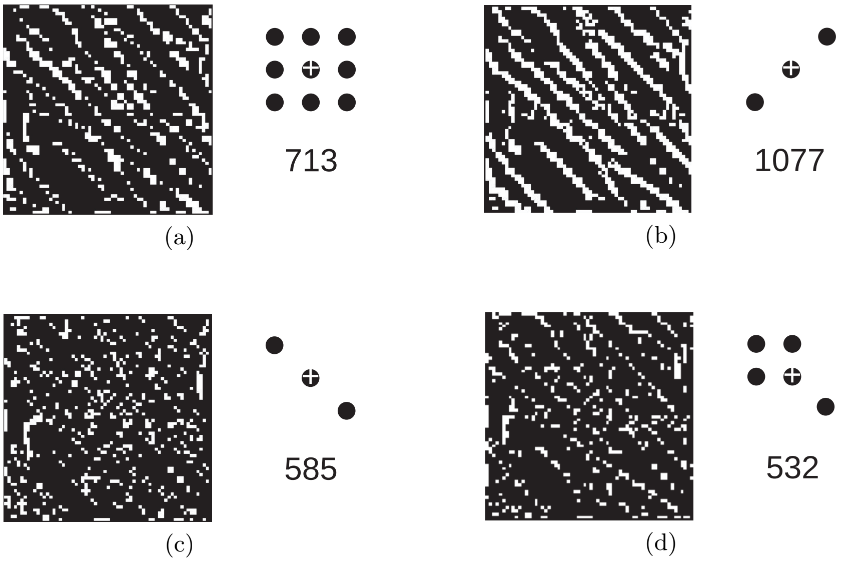

Figure 3. Result of primitive description.

element derived by our method within 5 × 5-pixel square is shown in Fig. 5 with the resultant skeleton. The optimal structuring element is like an oval. The primitive is described in the sense that the resultant structuring element is similar to the primitive in average.

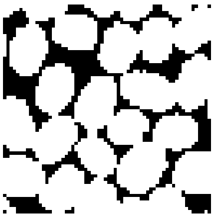

Figure 4. Another example texture.

Figure 5. Result of primitive description.

4. Conclusions

In this paper, we have proposed a novel method of the texture primitive description. We have employed the property that the skeleton of an object is reduced to one point when the structuring element used for the skeletonization is exactly homothetic to the object. We have minimized the number of pixels contained in the skeleton of a texture using the simulated annealing, and have found the structuring element homothetic to the primitive of the texture. We have assumed the model that the target texture is composed of one primitive, however, no assumption on the sizing distribution of grains is necessary.

We have proposed the method only for binary textures and structuring elements. We are now working on the extension to grayscale textures and structuring elements. The morphological skeleton is easily extended to gray scale images. We experienced that the two stage optimization is effective for the optimization of a gray scale structuring element [3]; the shape is optimized and fixed at the first stage, and the gray scale values are optimized at the second stage.

We have restricted ourselves to the case where the texture is composed of only one primitive. Application of this method to multiprimitive textures is also an open problem, although we have proposed an analyzing method for multiprimitive textures [11]. Tuning of the parameters used in the simulated annealing is also an open problem.

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