NOLTA 2006
11-14 september
Bologna - ITALY

Unsupervised structuring element optimization of morphological opening for texture images

Atsusi Fujiki†, Akira Asano‡, and Mitsuji Muneyasu†

†Faculty of Engineering, Kansai University
Yamatecho 3-3-35, Suita, Osaka 564-8680, JAPAN
‡Graduate School of Engineering, Hiroshima University
Kagamiyama 1-7-1, Higashi-Hiroshima, Hiroshima 739-8521, JAPAN
Email: asano@mis.hiroshima-u.ac.jp

Abstract—A method of optimizing structuring elements of morphological openings for extracting structures of texture image is proposed. This method can adopt the structuring element to the microstructure of a texture image, even if it is corrupted by noise. This method is based on the property of the texture that it is composed of a repetitive appearance of the microstructure. The extraction ability from noisy images is improved by introducing limitations to the variance of the pixel values within the structuring element in the process of optimization, and iterating the optimization with modifying the size of the structuring element. As an example of the application, the effective noise removal is achieved by the opening using the estimated shape of the microstructure as the optimal structuring element.

1. Introduction

Opening is one of the most important image operations in the context of mathematical morphology [1–3]. Opening presents the composition of an image by the repetitive arrangement of a structuring element, which is a small object used as a probe. The significance of opening is its quantitativeness in the sizes of image objects. For example, a quantitative noise removal in images is achieved by opening in the sense that noise objects smaller than the structuring element are removed exactly.

Since the shape of structuring element appears directly in the result of opening, the opening using the structuring element resembling objects contained in the target image preserves the visual appearance of the whole image. Of course it is not generally possible to determine one typical object resembling various objects contained in an image. However, if the target image is restricted to a texture, we can derive a typical object representing the whole texture.

The shape of repetitively appearing objects in a texture should be preserved by the optimal opening for texture characterization, since this is the fundamental characteristic for representing the visual appearance of the target texture. In the case that opening is applied for noise removal, a structuring element not resembling the repetitively appearing objects causes in the output image undesired microstructures which are not related to the original image. The optimal opening described above is achieved by using this typical object as the structuring element.

Such structuring element can be estimated by using "Primitive, Grain, and Point Configuration (PGPC)" texture model [4] which we have proposed as a model of texture description. The PGPC texture model regards a texture as an image composed by a regular or irregular arrangement of *grains* that are much smaller than the size of image and resemble each other, and presents a method of estimating the fundamental object, called *primitive,* from which the grains are derived by a certain modification.

We propose in this paper a novel optimization method which can estimate the primitive even if the target texture is corrupted by noise. It utilizes the a priori knowledge that the extent of noisy pixels is smaller than the original microstructure of textures, and introduces limitations on the shape of structuring elements based on this knowledge in the process of optimization.

We show an experimental result of an application of the proposed method for noise removal from texture images. In our previous work [5], we proposed an optimization method of gray scale opening for noise removal of texture images using the primitive estimation based on the PGPC texture model. The previous method, however, requires an example of the noise-free image of the target corrupted texture. It does not need to be the exact noise-free version of the target corrupted texture because of the characteristics of the textures described above, but should be at least a different uncorrupted realization of the texture which has the same microstructure as the target corrupted image.

The estimation method proposed in this paper can optimize the structuring element for noise removal without any noise-free example, since the proposed method can estimate the texture primitive from a noisy texture image. The experiment shows that the proposed method is as effective as the method using the noise-free example.

2. PGPC texture model and basic primitive estimation procedure

2.1. Morphological size distribution

Opening of image X with respect to structuring element B means residue of X obtained by removing smaller structures than B. It indicates that opening works as a filter to distinguish object structures by their sizes. Let $2B$, $3B$, $\ldots$, be homothetic magnifications of the basic structuring element B. We then perform opening of X with respect to the homothetic structuring elements, and obtain the image sequence X_B, X_{2B}, X_{3B}, $\ldots$. In this sequence, X_B is obtained by removing the regions smaller than B, X_{2B} is obtained by removing the regions smaller than X_{2B}, X_{3B} is obtained by removing the regions smaller than $3B$, $\ldots$. If B is convex, it holds that $X \subseteq X_B \subseteq X_{2B} \subseteq X_{3B} \subseteq \ldots$. This sequence of opening is called granulometry [3].

We then calculate the ratio of the area (for binary case) or the sum of pixel values (for gray scale case) of X_{rB} to that of the original X at each r. The area of an image is defined by the area occupied by an image object, i. e. the number of pixels composing an image object in the case of discrete images. The function from a size r to the corresponding ratio is monotonically decreasing, and unity when the size is zero. This function is called size distribution function. The size distribution function of size r indicates the area ratio of the regions whose sizes are greater than or equal to r.

2.2. PGPC texture model

The PGPC texture model regards a texture as an image composed by a regular or irregular arrangement of objects that are much smaller than the size of image and resemble each other. The objects arranged in a texture are called *grains,* and the grains are regarded to be derived from one or a few typical objects called *primitives*. This model is based on the observation, suggested by Gestalt psychology, that a repetitive appearance of similar objects of a moderate size is organized to be a meaningful structure by the human cognitive process.

We assume here that the grains are derived from one primitive by homothetic magnification.We also assume that the primitive is expressed by a structuring element B, and let X be the target texture image. In this case, X_{rB} is regarded as the texture image composed by the arrangement of rB only. It follows that $rB-(r+1)B$ indicates the region included in the arrangement of rB but not included in that of $(r+1)B$. Consequently, $X_{rB}-X_{(r+1)B}$ is the region where r-size grains are arranged if X is expressed by employing an arrangement of grains which are preferably large magnifications of the primitive. The sequence $X - X_B$, $X_B - X_{2B}$, $\ldots$, $X_{rB} - X_{(r+1)B}$, $\ldots$, is the decomposition of the target texture to the arrangement of the grains of each size.

2.3. Basic primitive estimation procedure

Since the sequence can be derived by using any structuring element, it is necessary to estimate the appropriate primitive that is a really typical representative of the grains. We employ an idea that the structuring element yielding the simplest grain arrangement is the best estimate of the primitive, similarly to the minimum description length (MDL) principle [6]. The simple arrangement locates a few number of large magnifications for the expression of a large part of the texture image, contrarily to the arrangement of a large number of small-size magnifications. We derive the estimate by finding the structuring element minimizing the integral of $1-F(r)$, where $F(r)$ is the size distribution function with respect to size r. The function $1 - F(r)$ is 0 for $r = 0$ and monotonically increasing, and 1 for the maximum size required to compose the texture by the magnification of this size. Consequently, if the integral of $1 - F(r)$ is minimized, the sizes of employed magnifications concentrate to relatively large sizes, and the structuring element in this case expresses the texture using the largest possible magnifications. We regard this structuring element as the estimate of primitive.

We estimate the gray scale structuring element in two steps: the shape of structuring element is estimated by the above method in the first step, and the gray scale value at each pixel in the primitive estimated in the first step is then estimated. However, if the above method is applied to the gray scale estimation, the estimate often has a small number of high-value pixel and other pixels whose values are almost zero. This is because the umbra of any object can be composed by arranging one-pixel structuring element. This is absolutely not a desired estimate. Thus we minimize $1 - F(1)$, i. e. the residual area of X_B instead of the above method. Since the residual region cannot be composed of even the smallest magnification, the composition by this structuring element and its magnification is the most admissible when the residual area is the minimum.

The exploration of the structuring element can be performed by the simulated annealing, which iterates a modification of the structuring element and find the best estimate minimizing the evaluation function described in the above [4].

3. Primitive estimation from corrupted image

The primitive estimation method described in the previous section has been developed for noiseless texture images. If it is applied for texture images corrupted with noise, it does not work well, since noisy pixels are incorporated to the estimation and the estimated primitive includes undesired noisy pixels.

To avoid this problem and achieve the estimation from corrupted image, we introduce the following two modification into the estimation procedure: 1) Limitation of the variance within the structuring element, and 2) Iterative estimation with reducing the extent of the structuring element.

3.1. Limitation of the variance within the structuring element

The extent of noisy pixels is usually small, and the pixel values are often significantly different from the neighborhood pixel values of the noiseless image. Thus the primitive estimated from noisy images often has a pixel whose value is significantly and unnaturally different from the other pixels. To avoid the problem, the variance of the pixel values within the structuring element is limited in the optimization procedure.

A structuring element is generated at each iteration by the modification to the structuring element at the previous iteration. If the variance of the pixel values is larger than the threshold, this structuring element is discarded and another structuring element is newly generated. The threshold is relatively large at the beginning of the procedure to allow a large variance, and decreases along the progress of simulated annealing to make the structuring element converge to the appropriate one.

3.2. Iterative estimation with reducing the extent of the structuring element

The extent of the estimated primitive is important when the primitive is used as the structuring element for opening. The smaller the extent is, the higher the ability of preserving the details of the image is, but the lower the ability of noise removal is. Thus we introduce an iteration of the estimation with reducing the extent of the primitive, to find the optimal extent.

After the initial estimation, the primitive is estimated again using the opened noisy image by the previously estimated primitive and reducing the extent of the primitive and the limitation of the variance. These estimations are iterated to obtain the final estimate.

4. Experiments

4.1. Parameter settings

We use 8-bit gray scale texture images of 64×64 for the experiments. Example texture images are shown in Fig. 1.

The estimation by simulated annealing has a parameter called *temperature* that controls the probability where the modification of structuring element is accepted even if the evaluation function is increased by this modification. The temperature at the ith iteration in one optimization process, denoted T_i is defined so that acceptance probability $P(\Delta IF)$ is as follows:

$$P(\Delta IF) = \frac{1}{1 + \exp(\frac{\Delta IF}{T_i})}, \quad (1)$$

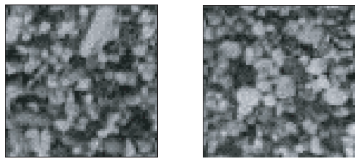

Figure 1: Example images used for the experiment.

where ΔIF is the increment of the evaluation function. The initial temperature T_0 is determined by setting the acceptance probability P at the initial state to 0.35 and the following calculation:

$$T_0 = \frac{\Delta IF}{\log(\frac{1}{P} - 1)}. \quad (2)$$

The temperature decreases following the iteration, as follows:

$$T_{i+1} = 0.98 T_i. \quad (3)$$

The number of iteration is fixed to 1000 times in our experiments.

The initial structuring element is set to the cross-shaped one of 9 pixels, and the initial pixel values are set to 50 at all the pixels. The number of pixels is fixed to 9 at the initial estimation, and the estimation procedures are repeated with decreasing the number of pixels by one, and the estimate where the number of pixels is 5 is regarded as the final estimate of the primitive. The final number of pixels is set to 5 based on a preliminary experiment. The threshold used for the limitation of variance is set to 500 when the number of pixels is 9, and decreases by 100 as the number of pixel decreases by one.

4.2. Results of optimization

We show here experimental results for noisy images corrupted by impulsive noise of positive values to present the effectiveness of the optimized opening. The pair of Fig. 1 (a) and (b) is extractions of different parts from the same texture. Corrupted images are generated by selecting 1000 pixels randomly in Fig. 1 (a) and adding random positive values to the selected pixels. If the noisy pixel value is larger than 255, it is replaced with 255.

Figure 2 shows the result of noise removal from the corrupted image generated from Fig. 1 (a). The estimation was tried five times, and the best results are shown here. The results by the opening optimized by our previous method [5] using noncorrupted images, and median filter, switching median filter, PSM filter [7] as typical image filter for impulsive noise, are shown for comparison. Figure 1(b) is used for the estimation by our previous method. The thresholds for the switching median filter and PSM filter are determined to yield the best results by repetitive experiments.

The mean-square-error in comparison with the original image (a) is shown at each image in Fig. 2. These results

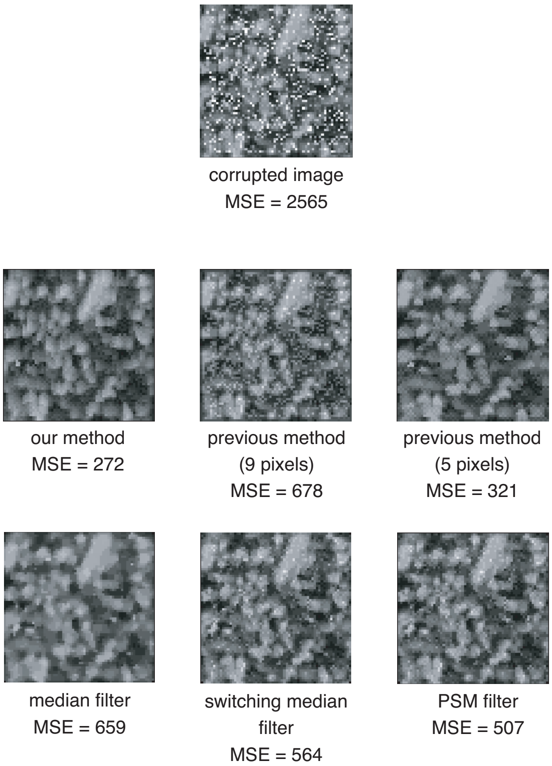

Figure 2: Results by the optimal opening for the noisy image generated from Fig. 1(a).

show that the proposed method has achieved the equivalent ability of the structure extraction and noise removal to our previous method without noncorrupted example images. They also show that our optimized opening has higher ability than the typical noise removing filters. It suggests that these typical filters degrade microstructures in texture images.

5. Conclusions

This paper has proposed the optimization method of morphological opening for extracting textural structures from noisy texture images. This method achieves the optimization with the target noisy image only and without any noncorrupted example by introducing an appropriate restriction. It is shown by an experiment of impulsive noise removal in texture images that the method is more effective than typical noise removing filters.

The purpose of our method is currently limited to the extraction of structures in texture images. The fundamental characteristic of the texture used in this method is, however, that the image is composed by the repetitive appearance of similar grains derived from a primitive, which is described by the PGPC texture model. It indicates that the method can be applied to general images other than textures, if the images can be transformed to those having the property. We are now working on this.

References

[1] J. Serra, Image analysis and mathematical morphology, Academic Press, 1982.

[2] P. Soille, Morphological Image Analysis, 2nd Ed., Springer, 2003.

[3] H. J. A. M. Heijmans, Morphological Image Operators, Academic Press, 1994.

[4] A. Asano, T. Ohkubo, M. Muneyasu, and T. Hinamoto, "Primitive and Point Configuration texture model and primitive estimation using mathematical morphology," Proc. 13th Scandinavian Conf. on Image Analysis, Göteborg, Sweden; Springer LNCS vol. 2749, pp. 178–185, Jun. 2003.

[5] A. Asano, Y. Kobayashi, C. Muraki, and M. Muneyasu, "Optimization of gray scale morphological opening for noise removal in texture images," *Proc. 47th IEEE International Midwest Symposium on Circuits and Systems (MWSCAS2004),* vol. 1, pp. 313–316, Jul. 2004.

[6] P. Grünwald, "A Tutorial introduction to the minimum description length principle," In: Advances in Minimum Description Length: Theory and Applications (P. Grünwald, I. J. Myung, M. Pitt, Ed.), MIT Press, 2005.

[7] Z. Wang and D. Zhang, "Progressive Switching Median Filter for the Removal of Impulse Noise from Highly Corrupted Images," IEEE Trans. Circuites & System II: Analog & Digital Signal Processing, Vol. 46, No. 1, pp. 78–80, Jan. 1999.