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An Unsupervised Design Method for Weighted Median Filters Using Genetic Algorithm

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***Abstract*—Estimation of a suitable window shape and appropriate weights in weighted median filters is one of important problems. Hitherto, an unsupervised design method for the filters has been proposed. In this method, a simulated annealing (SA) is applied to design the optimal filter for texture images corrupted by impulse noise, and it has been shown to perform as equally well as a supervised technique. In this paper, we apply a genetic algorithm (GA) and adopt Rank-Ordered Logarithmic Difference (ROLD) statistics as objective function to design a weighted median filter. Through experiments, it was shown that our new approach outperformed compared to conventional methods.**

I. Introduction

Images are often corrupted by impulse noise due to a noisy circuit or channel transmission errors. To remove impulse noise, many filters based on median filters have been proposed [1], [2]. Switching median filters [3] and weighted median filters [4] are typical one of such filters. The weighted median filter has the weight for each input in the filter mask. This filter can improve the estimation accuracy by given a larger weight to the sample to be processed than weights for other samples in the filter mask. In addition, the preservation of the detail is improved by adjusting its window shape for features of image. Therefore, how to decide the shape of the filter mask and weights is very important, especially for texture images, because a texture is composed of a lot of patterns which resemble each other, the relationship between the shapes of the pattern and the filter mask is affected to the noise reduction performance.

In our previous work, we proposed design methods for weighted median filters for texture images and adopted Simulated Annealing (SA) [5] as an optimization method [6], [7]. The proposed method in reference [7] is an unsupervised design method for the window shape and weights of the weighted median filter, which does not require any training images. This proposed technique adopts rank-order absolute differences (ROAD) statistic [8] to the objective function, and it performs as well as a supervised method [6].

In this paper, we adopt Genetic Algorithm (GA) [9], which is one of probabilistic optimization methods, and improve the design of weighted median filters. Our new proposed method is also an unsupervised design method. Just like the conventional SA approach, first GA estimates the window shape. After the optimization of the mask shape, the weights of the filter are also estimated by the same technique. Rank-Ordered Logarithmic Difference [10] statistic is used for both the measure of the filter performance and the construction of the objective function in the optimization by GA. Through experiments, we show the effectiveness of our new proposed technique.

II. Weighted Median Filters

For simplicity, one dimensional case of weighted median filters is introduced. Consider an input vector $\boldsymbol{X}=\{X_1, \cdots, X_{2N+1}\}$ and a weighted vector $\boldsymbol{W}=\{W_1, \cdots, W_{2N+1}\}$ composed of non-negative integer elements. Then, an output Y of a weighted median filter is given as (1) where MED means the median operation.

$$\begin{aligned} Y &= MED\{W_1 \diamond X_1, W_2 \diamond X_2, \cdots, W_{2N+1} \diamond X_{2N+1}\} \\ &= MED\{\underbrace{X_1, \cdots, X_1}_{W_1}, \underbrace{X_2, \cdots, X_2}_{W_2}, \\ &\qquad \cdots, \underbrace{X_{2N+1}, \cdots, X_{2N+1}}_{W_{2N+1}}\} \end{aligned} \tag{1}$$

By assigning a larger weight for an important signal, the value of signal or near value of it is output from the median operation due to high duplication of the signal [1].

When the weight in the above expression is extended to a real value, an output is calculated as follows. First, the elements of the input vector $\boldsymbol{X}$ is sorted in the ascending order. In the sorted sequence, let the ith largest element be $X(i)$ and the corresponding weight be $W(i)$. Then, $Y = X(k)$ is output, where k satisfies

$$k = \min_j \Big\{ j \Big| \sum_{i=1}^{j} W(i) > \frac{1}{2} \sum_{i=1}^{2N+1} W(i) \Big\}. \tag{2}$$

It is known that the output of the weighted median filter is equivalent to the value obtained by minimizing the function (3).

$$\Phi(\beta) = \sum_{i=1}^{N} W_i |X_i - \beta| \tag{3}$$

III. Proposed method

In this study, we discuss an unsupervised design method of filter for texture images. Pixel values of texture images are randomly arranged in local region; however, in global region, they have some pattern according to some rule [11]. Texture images include a lot of edges and details in the whole image, comparing with ordinary images. The processing by the conventional median filter to them might degrade the detail of the texture. Moreover, it might be difficult for the switching median filter to precisely identify the impulse noise in the texture image.

In this paper, we propose a new design method of a suitable window shape and appropriate weights in weighted median filters for texture images corrupted by impulse noise. Here, we apply a GA to optimize filters.

We adopt the model below similar to conventional methods [6], [7].

$$x(i,j) = \begin{cases} x_o(i,j) + l & \text{prob. } p \\ x_o(i,j) & \text{prob. } 1-p \end{cases} \tag{4}$$

Here, $x_o(i,j)$ indicates the pixel values of original image, and l means a non-negative integer with uniform distribution. $x(i,j)$ is rounded to 255 if $x_o(i,j)+l$ exceeds 255.

Just like the conventional SA approach [7], the window shape and the filter weights corresponding to the shape of the texture are estimated separately and no training image is required. The ROLD [10] statistic is used for both the measure of the filter performance and the construction of the objective function in the optimization by GA. We start from the estimation of the window shape. This is equal to the design for the window shape of median filters. This design might be considered as a part of the weight design, however the two step design process for the window shape and the weights is an efficient way for the whole design process. Since the shape of the window has a strong influence to the result for texture images and its suitable design can reduce the candidates to be evaluated, it is directly related to the reduction of the computational burden of the design process.

A. Application of GA

GA is among the most effective approximation algorithm for optimization problems. It is one of direct search methods that use an objective function value directly instead of evaluating features of functions, such as gradient. GA advances its search using multiple search points, which easily enables extension of a design into a multi-objective design. Therefore, GAs are applicable to a wide range of problems and have found many applications in combinatorial problems which have problem-specific structure and complex constraints like the design of weighted median filters.

The optimization procedure of GA for the estimation of window shapes or weights is as follows. Here, the generation alternation model based on Elitist Recombination (ER) model [12] is adopted.

Procedure of GA

Step 0/Initialization/
Generate the initial population $P(0)$ composed of N_{pop} random solutions (window shapes or weights), individuals, and evaluate them. The generation t=0.

Step 1/Selection for reproduction/
For generating offspring, N_{pop}/2 pairs of parents are randomly sampled without replacement from the current population $P(t)$.

Step 2/Offspring Generation/
Set the next population $P(t+1) = \phi$ and apply following procedures to each pair (p_1, p_2).

- (a) /Crossover/ Apply N_{cross} times of crossover operator to parents p_1 and p_2 and generate N_{cross} offspring.
- (b) /Mutation/ Apply a mutation operator to each offspring probability P_m.
- (c) /Selection for survival/ Select two best individuals from the family $F(p_1, p_2)$ consisting of p_1, p_2 and their offspring, and add them into $P(t+1)$.

Step 3/Check of the terminal criterion/
Go to Step 1 and set $t = t+1$ until some terminal criterion is satisfied, e.g., generations and/or the number of evaluations.

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1) Initialization of Individual: A solution, called an $individual$, in GA is a window shape possessing weights, which is desired to be formed randomly but spreading around the processing point. In generating the initial population, each initial individual is constructed with n elements, keeping the processing point the center as shown in Fig. 1. At each step of adding the elements to construct the window shape, one of neighboring positions (the dark gray region in the figure) of the intermediate shape is randomly selected. Due to this procedure, any split-off element does not occur. The position to be added is restricted in the 9x9 window and the number of elements of shape is fixed to nine.

For optimizing weights, a random value of the range [1, w_{max}] is assigned to each element of the window after optimizing the window shape as illustrated in Fig. 2. Here we set w_{max}=4.

2) Crossover and Mutation: Here, the crossover described below is applied to parents p_1 and p_2 for both optimization. Fig. 3 illustrates an example of this crossover on optimization of the window shape.

Procedure of Crossover

Step 0 Put the center element selected from p_1 or p_2 to the child.

Step 1 Select four elements except for the processing point from p_1 and copy them to the child.

Step 2 Select the other four elements except for the processing point from p_2 and copy them to the child.

As shown in Fig. 3, this crossover yields the child which has split-off points; however, no modification such as concatenation of them is applied to increase freedom of search of GA. In this crossover, only one child is generated from parents; nevertheless, the generation alternation model we adopt here applies the crossover more than once to each parent for reproduction and the population size keeps equal through the generation.

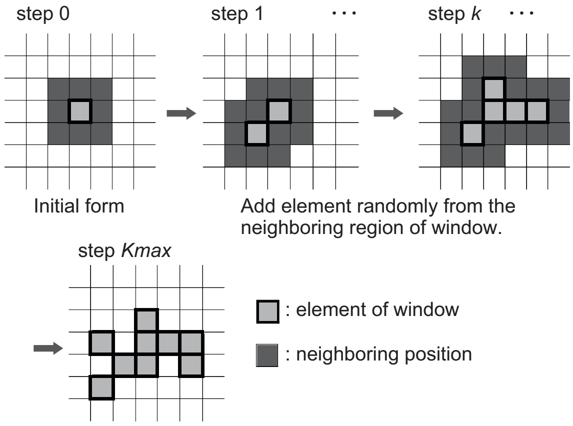

Fig. 1. Aspect of initialization of an individual (In shape optimization)

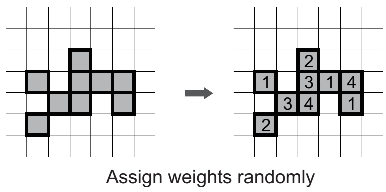

Fig. 2. Aspect of initialization of an individual (In weight optimization)

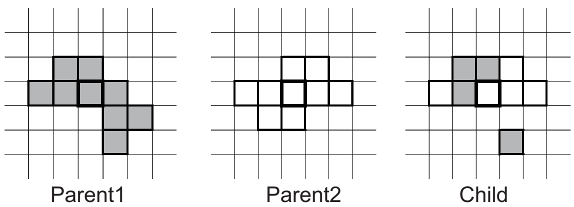

Fig. 3. Aspect of crossover method

In the mutation operator for optimizing the shape, the window shape is slightly modified. Only one element from the window shape is randomly selected and it randomly moves to another position. For optimizing the weights, one element is randomly selected from the window and assigned a random value of [1, w_{max}].

B. Design of Objective function

1) ROLD Statistic: In this approach, we adopt ROLD statistic to detect corrupted pixels by impulse noise.

Let a processing point and its neighborhood be $x = (x_1, x_2)$ and $\Omega_x(N) = \{x+(i,j) : -N \leq i, j \leq N\}$, respectively. The number of pixels in the neighborhood is (2N+1)x(2N+1). At each pixel in an image, ROLD statistic is defined as

$$ROLD_m(x) = \sum_{i=1}^{m} r_i(x) \tag{5}$$

where $r_i(x)$ means the ith smallest $d_{x,y}$ which is defined by

$$d_{x,y} = \{1 + \max\{\log_a |(u_x - u_y)/255|, -b\} \times 255 \tag{6}$$

where u_x and u_y denote the pixel values of the processing point and a neighborhood point of $\Omega_x(N)$. In (6), a and b are set to 2 and 5, respectively [10].

In conventional SA approach, ROAD was applied on the assumption that a pixel seems to be corrupted when its pixel value is considerably different among its neighborhood. However, under corrupting by impulse noise described by a uniform distribution, it is difficult for ROAD statistic to detect minute difference between corrupted and non-corrupted pixels. ROLD is an improved statistic of ROAD and enhances the difference between them by taking logarithms of absolute difference.

2) Estimation of the Window Shape: There are two intentions in the window optimization; one is the reduction of impulse noise and the other is the preservation of texture patterns. The reduction of impulse noise can be achieved to select an appropriate window size according to the probability of the noise. For the preservation of texture patterns, the selection of the window shape which exactly represents the direction and shape of the texture should be required. Therefore the latter is very important to decide the selection of the objective function.

To satisfy this requirement, the number of pixels which have the similar values in the estimated shape of the windows becomes a criterion, since it is considered that the iteration of specified patterns in the texture makes some shape with similar pixel values. Therefore for the optimal window shape, its value of (3) becomes lower, because the similar value of pixels in the window leads a low absolute difference among pixels.

The ROLD statistic can be satisfied with the above requirement and can evaluate the performance of the estimated window shape. In this paper, we assume that non degraded pixels have about half the number of pixels of which values are almost same in its neighborhood. As the evaluation function, the following ROLD statistic is adopted:

$$ROLD_4(x) = \sum_{i=1}^{m} r_i(x) \tag{7}$$

If this value derived from (7) is small, there are at least four pixels which have the similar value with the processing point and the value of (3) can also become small. The corrupted pixels are almost eliminated from the calculation of (7) and the effect of them can be ignored if the processing point is not corrupted by noise. The minimization of (7) can then estimate the optimal window shape which corresponds to the direction and shape of the texture pattern.

As a result, the total sum of (7) for each processing point is adopted as the objective function for optimizing the window shape:

$$F_1 = \sum_{j=1}^{M} R_j(x) \tag{8}$$

where M means the total number of the pixels times (1 - p), where p is the estimated probability of noise, and $R_j(x)$ the jth smallest value of the arrangement of $ROLD_4(x)$ sorted in the ascending order.

3) Estimation of weights: After the optimization of the window shape, the weights in the weighted median filter which has the optimized window shape are estimated. As initial weights, random values from 1 to w_{max} are assigned to each element of the window. Under these conditions, GA similarly advances the search and the best weights and window shape are then optimized.

In this estimation, another objective function is constructed for more consideration of the preservation of the texture pattern. If the optimal shape of the window is obtained, the impulse noise is almost reduced. The weights can be then used for the preservation of the original image. In this case, we separate the objective functions for non-corrupted part and the corrupted part. For non-corrupted parts, the mean square error (MSE) between the pixel value of the processing point and the output of the estimated filter is adopted. On the other hand, for the corrupted parts, some estimation of the true pixel value for the processing point is required. Here, the median value of five pixels which have the smaller value than the other in the window is used for the estimation, because the optimized window composes of the pixels which have the almost same value. Using this estimation as the value of the processing point, the MSE can be also calculated. Moreover, in the discrimination of the nature of the pixels, the ROLD statistic is adopted and the threshold value Th is decided experimentally.

From the above discussion, the objective function is defined as follows. For non-corrupted parts decided by ROLD, the value of objective function is defined as

$$F_2 = \sum \Big| z(i,j) - y(i,j) \Big|^2 \tag{9}$$

where $z(i,j)$ and $y(i,j)$ indicate the value of the processing point and the output of the filter, respectively.

The value of objective function of corrupted parts is defined as

$$F_2' = \sum \Big| m(i,j) - y(i,j) \Big|^2 \tag{10}$$

where $m(i,j)$ indicates the median value of five pixels which have the smaller value than the other in the window. As a result, we adopt

$$F = \frac{1}{L}\Big(\sum \Big| z(i,j) - y(i,j) \Big|^2 + \sum \Big| m(i,j) - y(i,j) \Big|^2 \Big) \tag{11}$$

as total objective function, where L means the total number of the pixels in an image.

IV. Numerical Experiments

To show the effectiveness of the proposed method, the design results by conventional method of SA and the proposed method are compared. Six kinds of images; $bulap$, $cork$, fur, $matting$, $sand$ and $water$ of Brodatz textures [13] were used for the examination. The size of each image was 64x64 and its gray levels 256. The probability of the impulse noise was 0.2.

For experiments, the population size N_{pop} was set to 50, and a search was terminated after 40 generations of GA for both optimization. Each pair of parents for the crossover generated 10 offspring and the mutation rate was set to $1/N_{pop}$. Th of ROLD was set to 250 for $bulap$, $sand$ and $water$, 500 for others.

In the conventional method, for the estimation of the window shape, the initial temperature was 0.3 and the number of iteration was 350. For the estimation of the weights, the initial temperature was 0.1 and the number of iteration was 200. The cooling rate was 0.96 for both optimization. The threshold value of ROAD was 120 which was decided experimentally.

Table I shows the best value of MSE between the processing result and the original image, the averaged MSE (avg.) and the standard deviation of MSE (std.) out of 20 trials. The estimation results with the best solutions are shown in Figs. 4-9. The colored element in these figures means the processing point in the window.

TABLE I
Search Performance of SA and GA: The best, average and standard deviation of MSE out of 20 trials

Instance	Degraded	SA			GA		
	Image	best	avg.	std.	best	avg.	std.
burlap	3051.2	1316.8	1530.0	110.0	**842.6**	1510.5	341.0
cork	1612.4	886.4	983.2	58.0	**625.2**	960.5	91.2
fur	799.2	404.8	438.4	22.4	**255.7**	341.7	23.1
matting	1530.4	793.6	889.6	54.4	**706.7**	781.5	54.3
sand	1778.0	430.8	455.6	14.0	**298.1**	519.9	59.8
water	1860.4	429.2	461.6	22.8	**346.9**	481.4	40.9

From these results, the proposed GA almost outperforms to the conventional method of SA from the perspective of the best MSE. Averages of MSE become worse in a few texture. This is because GA designs the shape of window with eight free elements while SA fixes five points around the center. These would be improved by using a large population size and a generation alternation model with advanced diversity.

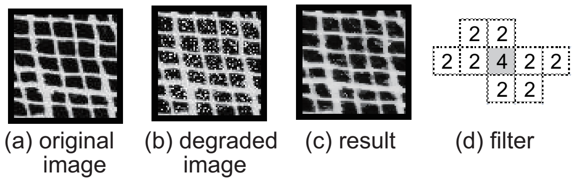

Fig. 4. The processing result obtained by GA (burlap)

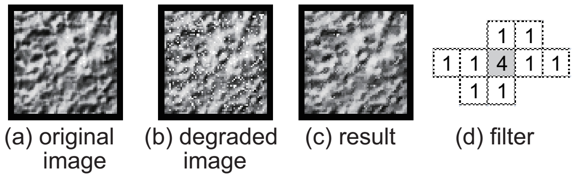

Fig. 5. The processing result obtained by GA (cork)

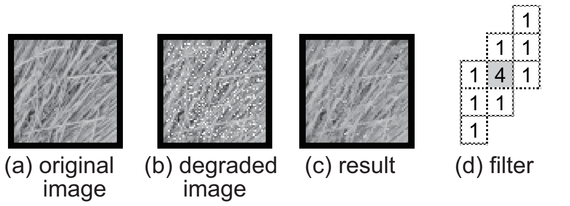

Fig. 6. The processing result obtained by GA (fur)

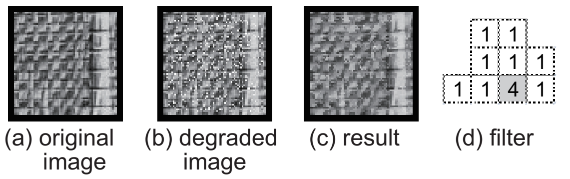

Fig. 7. The processing result obtained by GA (matting)

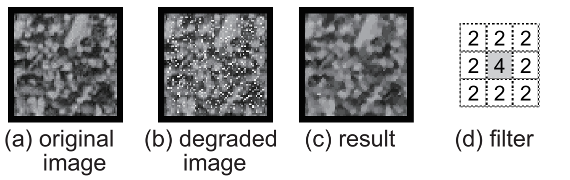

Fig. 8. The processing result obtained by GA (sand)

V. Conclusions

This paper has proposed a new unsupervised design method for weighted median filters using GA. The proposed method is based on the ROLD statistic for the objective function and the adoption of this makes the unsupervised design possible. The proposed method is most effective as it can optimize by using only the degraded image like the conventional approach of SA. From the experimental results, the window shape and weights suitable for the texture pattern can be designed and its performance has been almost superior to the conventional method.

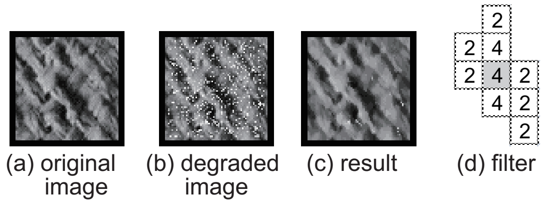

Fig. 9. The processing result obtained by GA (water)

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