An Unsupervised Optimization of Structuring Elements for Noise Removal Using GA

Hiroyuki Okuno* Yoshiko Hanada* Mitsuji Muneyasu* and Akira Asano†
* Faculty of Engineering Science, Kansai University, Japan
{hanada, muneyasu}@ipcku.kansai-u.ac.jp
† Graduate School of Engineering, Hiroshima University, Japan
asano@mis.hiroshima-u.ac.jp

***Abstract*—To recover texture images from impulse noise by the opening operation which is one of morphological operations, an suitable structuring element (SE) has to be estimated. Hitherto, an unsupervised design method for the SEs has been proposed, and it has adopted Simulated Annealing (SA) as an optimization method. In this conventional approach, SEs are designed under a neighborhood structure which keeps the size of shape, and the size is gradually reduced in the search process. Due to this, the search space is restricted. In this paper, Genetic Algorithm (GA) which can search effectively on wide search spaces is applied and the size of shape is included in design variables. Through experiments, it is shown that our new approach outperforms the conventional method.**

I. Introduction

Mathematical morphology is a fundamental framework of image manipulation, and a wide range of nonlinear image processing filters can be unified into the framework of mathematical morphology [1], [2], [3]. *Opening* and *closing*, which is the dual of opening, are typical morphological operations, and fundamental morphological filters that have idempotence. They are used for various methods of noise reduction, object extraction, etc.

Mathematical morphological operations manipulate an image with a small object called *structuring element* (SE), which is equivalent to the window of image processing filters. Opening composes the resultant image object by arranging the SE inside a target object and removes residual regions that are too small to locate the SE inside. The significance of opening is its quantitativeness in the sizes of image objects. The impulse noise removal by opening achieves a quantitative operation in the sense that noise objects smaller than the SE are removed exactly.

Since opening composes an image by repetitively locating a SE, its shape and gray scale distribution directly appear in the resultant image. In the case that the SE is inappropriate to the image, it causes appearance of undesired microstructures which are not related to the original image. These problems can be avoided by the usage of an appropriate gray scale SE that resembles the objects in the target image. Thus determination of the shape and gray scale distribution of the SEs is an important problem.

*This work was partially supported by Grant-in-Aid for Scientific Research (C), 20560357.

In our previous work, we proposed design methods of SEs for texture images [4], [5]. These optimization methods estimate suitable SEs for gray scale opening for noise removal of texture images based on the *Primitive, Grain, and Point Configuration (PGPC) texture model* [6] which regards a texture as an image composed by a regular or irregular arrangement of fundamental objects that are much smaller than the size of image and resemble each other. Among them, the proposed method in reference [5] is an unsupervised design method for the shape and gray scale pixel values of SEs, which does not require any training images. This proposed technique adopts the *size distribution function* to construct the objective function and uses Simulated Annealing (SA) [7] as an optimization method. It performs as well as a supervised method; however, it designed under a neighborhood structure which keeps the size of shape, which restricts the search space and decreases the possibility to discover better solutions.

In this paper, we adopt Genetic Algorithm (GA) [8], which is one of probabilistic optimization methods, and improve the unsupervised design method of SEs. In this approach, the size of shape is included in design variables to obtain wide variety of SEs. Just like the conventional SA approach, first GA estimates the shape. After the optimization of shape, the pixel values of SEs are also estimated. Through experiments, we show the effectiveness of our new proposed technique.

II. Mathematical Morphology

A. Mathematical Morphology and Opening

In the context of mathematical morphology, an image object is defined by a set. In the case of binary images, this set contains the pixel positions included in the object, i.e., those of white pixels. In the case of gray scale images, an image object is defined by an *umbra* set. If the pixel value distribution of an image object is denoted as $f(\boldsymbol{x})$ where $\boldsymbol{x} \in R^2$ is a pixel position, its umbra $U[f(\boldsymbol{x})]$ is defined as follows:

$$U[f(\boldsymbol{x})] = \{(\boldsymbol{x}, t) \in R^3 | f(\boldsymbol{x}) \geq t > -\infty\} \tag{1}$$

Consequently, when we assume a solid of which support is the same as a gray scale image object and height of which at each pixel position is the same as the pixel value at this position, the umbra is equivalent to this solid and the whole volume below this solid within the support.

Another object, called *structuring element* (SE), is defined in the same manner above. The SE is equivalent to the window of image processing filters, and supposed to have much smaller extent than the image object.

In the case of a binary image and a SE, *opening* of an image object X with respect to a SE B, denoted X_B, has the following properties:

$$X_B = \{B_Z | B_Z \subset X, z \in R^2\} \tag{2}$$

where B_Z indicates the translation of B by z.

Here, we concentrate ourselves to opening in the explanation in this and the next section since the operations on *closing* are regarded as the dual of opening.

In the case of the gray scale image and a SE, opening is similarly defined by replacing the sets X and B with their umbrae, respectively, and supposing that $z \in R^3$.

This property indicates that opening is the regeneration of an image by arranging the SE, and removes smaller white regions in binary case or smaller regions composed of brighter pixels than its neighborhood in gray scale case than the SE. Since opening eliminates smaller structures and smaller bright peaks than the SE, it indicates that opening works as a filter to distinguish object structures by their sizes.

B. PGPC Texture Model and Primitive Estimation

The *Primitive, Grain and Configuration* (PGPC) model [6] represents a texture as an image composed by a regular or irregular arrangement of objects which are much smaller than the size of image. The objects arranged in a texture are called *grains*, and the grains are regarded to be derived from one or a few typical objects called *primitives*.

We assume that the primitive is expressed by a SE. We also assume here that the grains are derived from r times of homothetic magnification of one primitive. The grain that is a result of homothetic magnification, rB, is defined as (r-1) times of *Minkowski* set additions between SE B and another small element. In this case, X_{rB} is regarded as the texture image X composed by the arrangement of rB only. It follows that rB-(r+1)B indicates the region included in the arrangement of rB but not included in that of (r+1)B. Consequently, X_{rB}-$X_{(r+1)B}$ is the region where r size of grains are arranged if X is expressed by employing an arrangement of grains which are preferably large magnifications of the primitive. The sequence $\{X$-X_B, X_B-X_{2B}, $\cdots$, X_{rB}-$X_{(r+1)B}$, $\cdots\}$ is the decomposition of the target texture to the arrangement of the grains of each size.

Since the sequence can be derived by using any SE, it is necessary to estimate the appropriate primitive that is a really typical representative of the grains. We employ an idea that the SE yielding the simplest grain arrangement is the best estimate of the primitive, similarly to the minimum description length (MDL) principle. The simple arrangement locates a few number of large magnifications for the expression of a large part of the texture image, contrarily to the arrangement of a large number of small-size magnifications.

III. Proposed Method

In this study, we discuss an unsupervised design method of SEs for noise removal for texture images using GA. The shape of SE and its pixel value of each element are estimated separately and no training image is required.

There are several impulse noise models for images. Here, we adopt the model below.

$$x(i,j) = \begin{cases} x_o(i,j) + l & \text{prob. } p \\ x_o(i,j) & \text{prob. } 1-p \end{cases} \tag{3}$$

Here, $x_o(i,j)$ indicates the pixel values of original image, and l means a non-negative integer with uniform distribution. $x(i,j)$ is rounded to 255 if $x_o(i,j)+l$ exceeds 255.

A. Application of GA

GA is among the most effective approximation algorithm for optimization problems. It is one of direct search methods that use an objective function value directly instead of evaluating features of functions, such as gradient. GA advances its search using multiple search points, which easily enables extension of a design into a multi-objective design.

In our approach, we start from the estimation of optimal shape of SE and then optimize the pixel values of each element of it.

The optimization procedure of GA for the estimation of shapes or pixel values is as follows. Here, the generation alternation model based on Elitist Recombination (ER) model [9] is adopted.

Procedure of GA

Step 0/Initialization/
Generate the initial population $P(0)$ composed of N_{pop} random solutions (shapes or pixel values), individuals, and evaluate them. The generation t=0.

Step 1/Selection for reproduction/
For generating offspring, N_{pop}/2 pairs of parents are randomly sampled without replacement from the current population $P(t)$.

Step 2/Offspring Generation/
Set the next population $P(t+1) = \phi$ and apply following procedures to each pair (p_1, p_2).

- (a) /Crossover/ Apply N_{cross} times of crossover operator to parents p_1 and p_2 and generate N_{cross} offspring.
- (b) /Mutation/ Apply a mutation operator to each offspring probability P_m.
- (c) /Selection for survival/
Select the best individual and another individual by a roulette selection from the family $F(p_1, p_2)$ consisting of p_1, p_2 and their offspring, and add them into $P(t+1)$.

Step 3/Check of the terminal criterion/
Go to Step 1 and set $t = t + 1$ until some terminal criterion is satisfied, e.g., generations and/or the number of evaluations.

---

1) Representation of Individual: Each SE is an individual of GA and represented with a two-dimensional binary array for optimizing a shape of SE. Fig. 1 shows an instance of shape that consists of n pixels randomly selected from a square 3x3 pixel. In this figure '1' indicates an element of the SE. In this representation, the size of shape is included in design variables. In this paper, each SE is formed in the range of 5 to 9 pixels selected from a square 9x9 pixel. Consequently, each individual of GA is expressed as binary string of length 81.

In a pixel value optimization, a random integer value of the range $[0,\ 127]$ is assigned to each element of the SE after optimizing the shape. Each pixel value is coded into a binary string of 7 bits. The chromosome length of individual is 7xn.

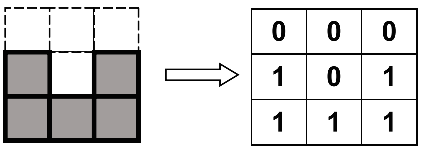

Fig. 1. Aspect of initialization of an individual

2) Crossover and Mutation: For estimation of the optimal shape, a mask array of $\{0,\ 1\}$ is generated randomly for each pair of parent, and uniform crossover (UX) is applied to the parents as shown in Fig. 2. In the UX, the element of parent p_1 is copied to the child c_1 when the value of mask is '1', while the element of child c_2 reflects the value of p_1 in the case of '0'.

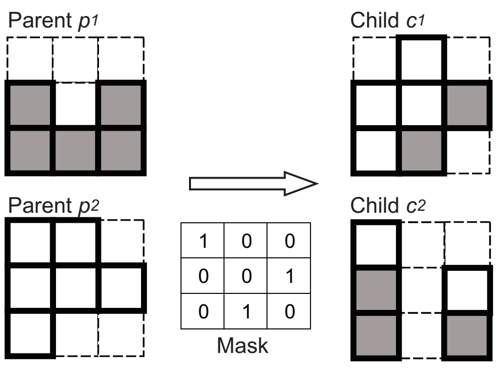

Fig. 2. Instance of uniform crossover

As a constraint for optimizing the shapes, an individual does not have split-off points. The offspring generated by UX do not necessarily satisfy this condition like c_2 shown in Fig. 2. In such a case, the individual takes a penalty in the evaluation. The generation alternation model we adopt here can reproduce a wide variety of offspring that inherit favorable characteristics of parents by applying the crossover more than once to each pair of parent. In the mutation, a random bit flip operator is applied to offspring.

For pixel values optimization, UX and the bit flip mutation are applied to binary strings.

B. Design of Objective Function

Here, we adopt a *size distribution function* [5] for constructing objective functions. Opening of image X with respect to SE B means residue of X obtained by removing smaller structures than B. We perform opening of X with respect to the homothetic SEs rB which are results of r times of homothetic magnification of B, and obtain the image sequence $\{X,\ X_B,\ X_{2B},\ \cdots,\ X_{rB},\ \cdots\}$. In this sequence, X_{rB} is obtained by removing the regions smaller than rB. We then calculate the ratio of the area (for binary case) or the sum of pixel values (for gray scale case) of X_{rB} to that of the original X at each r. The area of an image is defined by the area occupied by an image object, i.e., the number of pixels composing an image object in the case of discrete images. The function from a size r to the corresponding ratio is monotonically decreasing and it is unity when the size is 0. This function is called the size distribution function. The size distribution function of size r, $F(r)$, indicates the area ratio of the regions whose sizes are greater than r or equal to r.

In the conventional SA approach, the integral of 1-$F(r)$ is used as the objective function for optimization of both a shape and pixel values. Here, we minimize the integral of $F(r)$ for optimizing pixel values while minimization of the integral of 1-$F(r)$ is applied to the optimization of shape.

In this section, we confirm the validity of objective functions for evaluation of SEs and discuss difficulty of this design problem by investigating the correlation between a evaluation value and MSE.

1) Shape Optimization: We minimize the integral of 1-$F(r)$ to estimate the optimal shapes just like the conventional SA approach.

Fig. 3 shows the relation among evaluation values and MSEs between the original texture and a processing result of SE. These are distributions of 3000 random individuals, SEs, on *burlap (D103)*, *cloth (D101)*, *sand (D29)* and *straw (D49)* of Brodatz textures [10]. The size of each image is 64x64 and its gray levels 256. Corrupted texture images were generated by selecting 1000 pixels randomly.

In these figures, distribution which extends diagonally from the bottom left to the top right means that we can obtain closer processing results to original textures by optimization. From Fig. 3, minimization of 1-$F(r)$ is effective on burlap, sand, and straw. In the case of straw, distribution is separated to some parts, which indicates the fitness landscape of objective function is multimodal whose local optima are dispersed.

In contrast, the distribution on cloth spreads up and down, and the correlation between the evaluation value and MSE is poor. In this case, the processing result is not necessarily close to the original texture even though SE is optimized. Another suitable objective function should be designed to this case;

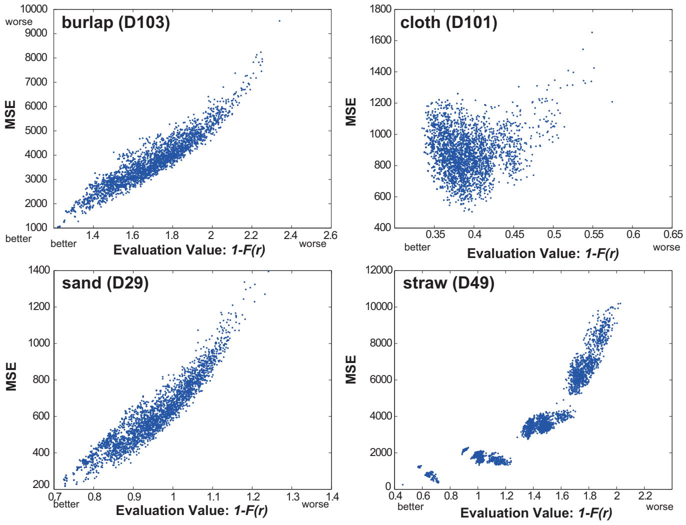

Fig. 3. MSE vs Evaluation Value in Shape Optimization

however, we adopt 1-$F(r)$ as the objective function on overall textures.

2) Pixel Value Optimization: We minimize the integral of $F(r)$ to estimate the optimal pixel values contrary to the shape optimization. This is because the positive correlation between evaluation values and preferable processing results can be observed by minimizing $F(r)$ as shown in Fig. 4.

These results are distributions of 3000 individuals whose shapes are optimized on *cork (D4)*, *marble (63)*, *matting (83)* and *water (37)*. Pixel values of each individual are randomly assigned in the range of [0, 127] and corrupted texture images were generated by selecting 1000 pixels randomly. Dashed line in these figures indicates the MSE of processing result of flat SE that has the optimal shape whose pixel values are 0.

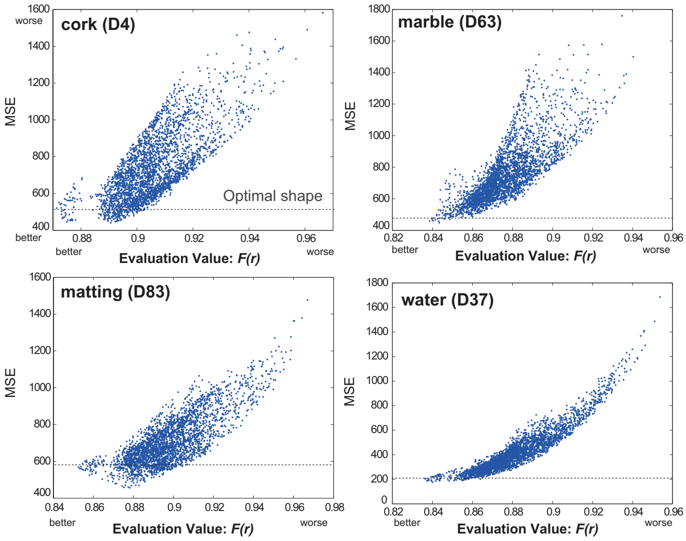

Fig. 4. MSE vs Evaluation Value in Pixel Value Optimization

From these results, importance of pixel value optimization is confirmed by comparing the processing results of SE of the optimal shape. Distributions of overall instances extend diagonally from the bottom left to the top right and we can confirm that closer processing results to original textures are obtained by optimization. Among them, we can see landscape of objective function on cork is multimodal. In the case of matting in which the minimum of evaluation value and that of MSE are out of synchronization, closer processing results to the original texture are not necessarily obtained even though we improve search mechanism of GA. In addition, wide variability on the same evaluation value is observed and the improvement of design of objective function is one important issues of the future work.

IV. Numerical Experiments

To show the effectiveness of the proposed method, the design results by conventional method of SA and the proposed method are compared. 10 kinds of images; $beans(D75)$, $burlap(D103)$, $cloth(D101)$, $cork(D4)$, $fur(D93)$, $marble(D63)$, $matting(D83)$, $sand(D29)$, $straw(D49)$ and $water(D37)$ of Brodatz textures [10] were used for the examination. The size of each image was 64x64 and its gray levels 256. We used corrupted texture images generated by randomly selecting 1000 pixels.

For experiments, the population size N_{pop} was set to 30, and a search was terminated after 30 generations of GA for both optimization. Each pair of parents for the crossover generated 30 offspring. The mutation rate was set to 0.01 for the shape optimization and 0.05 for the pixel value optimization. The procedure and parameters of conventional SA approach are described in the appendix A.

Table I shows the best value of MSE between the processing result and the original image, the averaged MSE (avg.) and the standard deviation of MSE (std.) out of 20 trials.

In this table, ”s/p” indicates predicted problem difficulties on both the shape optimization (s) and the pixel value optimization (p) based on the correlation between evaluation values and MSEs. ◯ indicates that the problem has positive correlation and GA would be effective. △ shows a separate but positive correlation, which means that fitness landscape of problem has distributed local optima. × indicates that no correlation is observed or there is a gap between the minimum evaluation value and that of MSEs.

For comparison, the best processing result of SE obtained by SA and the processing result of SE derived from the shape optimization are illustrated. The estimation results with the best solutions are shown in Figs. 5-9.

From Table I, the proposed GA approach outperforms the conventional method. From the average of MSE, the proposed GA obtains good solutions stably, though it searches larger space. In addition, the results of pixel value optimization performs equally or superior to the results of shape optimization only, and we can confirm the effectiveness of it in most texture images.

On the instances whose difficulties indicate ◯ or △, e.g., cork, fur, and marble, improvement is anticipated by using a large population size and other effective crossovers and

TABLE I
PROCESSING RESULTS: THE BEST, AVERAGE AND STANDARD DEVIATION OF MSE OUT OF 20 TRIALS OF GA

Instance name	s/p	Degraded Image	GA best	GA avg.	GA std.	SA best	Optimal shape
beans	×/△	2213	**705**	713.6	4.83	988	**705**
burlap	○/×	4210	**1026**	1034.4	4.06	1826	1046
cloth	×/○	3459	**967**	981.8	8.95	994	1040
cork	○/△	2271	516	528.8	4.88	626	**514**
fur	○/○	1244	**310**	316.6	2.44	626	**310**
marble	○/○	2624	473	476.1	1.49	**467**	477
matting	○/×	2014	**573**	578.2	2.91	591	581
sand	○/○	2565	**248**	253.1	1.73	272	255
straw	△/○	2846	**247**	251.5	2.16	326	254
water	○/○	2503	**195**	200.1	2.23	232	210

generation alternation model with advanced diversity. On the other hand, instances which have little correlation between evaluation values and preferable processing results, such as beans, require reconsideration of objective functions.

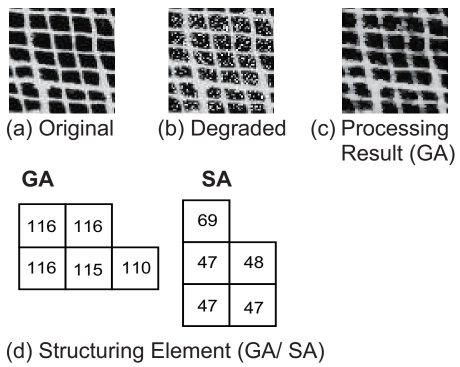

Fig. 5. Estimation of burlap by GA

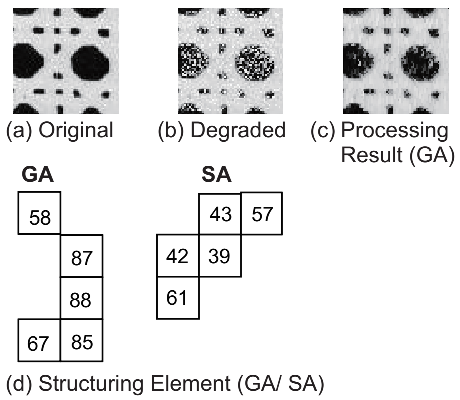

Fig. 6. Estimation of cloth by GA

V. CONCLUSION

This paper has proposed a new unsupervised design method of SEs for impulse noise removal for texture images using GA. We first confirmed the validity of using the size distribution function $F(r)$ as objective functions, and we adopted the integral of 1-$F(r)$ for the shape optimization, and the integral of $F(r)$ to optimize pixel values of SEs. We compared the proposed GA approach to the conventional SA approach by using several kinds of texture of which difficulties were different one another, and showed that the proposed GA almost outperformed the conventional method from the perspective of the best and the average of MSE. There are some texture where the size distribution function cannot work well, and improvement of design of objective function is one important issues of the future work.

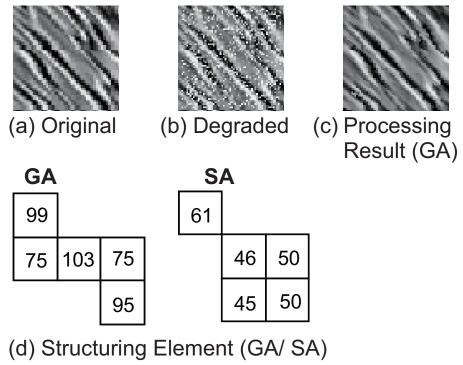

Fig. 7. Estimation of marble by GA

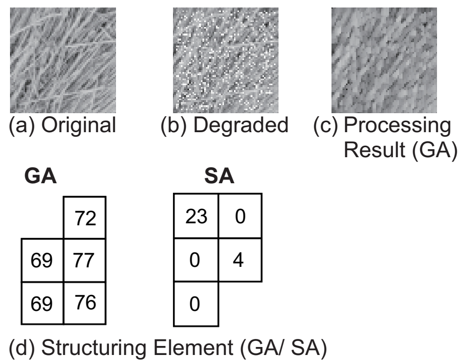

Fig. 8. Estimation of straw by GA

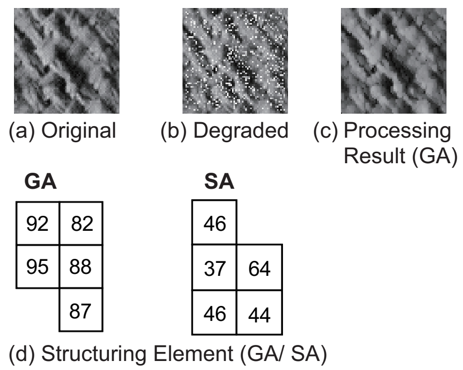

Fig. 9. Estimation of water by GA

REFERENCES

[1] J. Serra: Image analysis and mathematical morphology, Academic Press, 1982.
[2] H. J. A. M. Heijmans: Morphological Image Operators, Academic Press, 1994.

[3] P. Soille: Morphological Image Analysis, 2nd Ed., Springer, 2003.
[4] A. Asano, Y. Kobayashi, C. Muraki, and M. Muneyasu: Optimization of gray scale morphological opening for noise removal in texture images, Proc. 2004 IEEE International Midwest Symposium on Circuits and Systems, vol.1, pp.I-313-I-316 (2004).
[5] Atsusi Fujiki, A., Asano, A. and Muneyasu, M.: Unsupervised optimization of morphological filters for noise removal in texture images. Proc of SCIS & ISIS2006, pp. 1794–1799, 2006.
[6] A. Asano, T. Ohkubo, M. Muneyasu, and T. Hinamoto: Primitive and Point Configuration Texture Model and Primitive Estimation using Mathematical Morphology, Proc. Scandinavian Conference on Image Analysis 2003, pp.178-185, 2003.
[7] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi: Optimization by Simulated Annealing, Science, Vol. 220, No. 4598, pp. 671-680, 1983.
[8] Goldberg, D. E.: Genetic Algorithms in Search Optimization and Machine Learning, Addison-Wesley, 1989.
[9] D. Thierens and D. E. Goldberg. Elitist Recombination: an integrated selection recombination GA. Proceedings of the 1st IEEE Conference on Evolutionary Computation, pp. 508-512, 1994.
[10] P. Brodatz: Textures: A Photographic Album for Artists and Designers, Dover Pubns, 1999. http://www.ux.uis.no/ tranden/brodatz.html

APPENDIX

A. Estimation using SA

The estimation by simulated annealing (SA) has a parameter called *temperature* that controls the probability where the modification of SE is accepted even if the evaluation function is increased by this modification. The temperature at the k-th iteration in one optimization process, denoted T_k is defined so that acceptance probability $P(\Delta IF)$ is as follows:

$$P(\Delta IF) = \frac{1}{(1 + \exp(\Delta IF/T_k))} \tag{4}$$

where ΔIF is the increment of the evaluation function. The initial temperature T_0 is determined by setting the acceptance probability P at the initial state to 0.35 and the following calculation:

$$T_0 = \frac{\Delta IF}{\log(1/P - 1)} \tag{5}$$

The temperature decreases following the iteration, as follows:

$$T_{k+1} = 0.98 T_k \tag{6}$$

The number of iteration is fixed to 1000 times in our experiments.

The initial structuring element is set to the cross-shaped one of 9 pixels, and the initial pixel values are set to 50 at all the pixels. The number of pixels is fixed to 9 at the initial estimation, and the estimation procedures are repeated with decreasing the number of pixels by one, and the estimate where the number of pixels is 5 is regarded as the final estimate of the primitive. The final number of pixels is set to 5 based on a preliminary experiment. The threshold used for the limitation of variance is set to 500 when the number of pixels is 9, and decreases by 100 as the number of pixel decreases by one.