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Texture Analysis based on Mathematical Morphology and MDL Principle

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***Abstract*—A texture analyzing method based on a model that composes a texture of an arrangement of similar grains has been proposed. It estimates a typical shape representing the grains, called primitive, and the arrangement based on the model. This paper proposes a method of estimation based on mathematical morphology and the minimum description length (MDL) principle. This method can estimate the primitive of a suitable size without any unnatural assumption on the primitive. The effectiveness of the method is certified by an experiment of noise removal in texture images by morphological opening with the estimated primitive as the structuring element.**

I. Introduction

Texture analysis is an important topic of image sciences. Most of the conventional texture analyzing methods mainly aim at identification or discrimination of textures [1]. They calculate features of the target textures and discriminates textures according to the features.

On the contrary, this work provides a method of describing *structures* contained in textures. A texture is recognized as a repetitive pattern since the microstructure and its arrangement composing the texture are regarded as fundamental entities representing the texture. A significance is in the structure. This work describes the specific shape of the microstructures and expresses the characteristics of the target texture.

Our idea is similar to the method of statistical inference that assumes a model of probability distribution and estimates a parameter of the model using observations. We employ a model in which microstructures in a texture resemble but slightly differ from each other since the texture is generated by an arrangement of variations of a *primitive*. Texture analysis in our work is the estimation of the primitive. The variations are called *grains* in this model.

We call this model *PGPC (Primitive, Grain, and Point Configuration) texture model,* and have proposed methods of primitive estimation using morphological size distribution [2]. Size distribution decomposes a figure to a combination of homothetic magnifications of a given structuring element and expresses characteristics of the figure by the ratio of the region belonging to the magnification of each size [3]. Our conventional methods introduce an assumption to the size distribution of a texture based on the texture generation model, and describe the primitive by the optimal structuring element satisfying the assumption best.

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This idea, in other words, calculates the fitness of a structuring element to the grains contained in the target texture image, and searches the structuring element yielding the highest fitness. If we assume that the structuring element that is exactly the same as the target texture image itself, it completely fits the target texture. On the contrary, if we assume that the structuring element is of one pixel, it also completely fit the target texture. These structuring elements are not what we want now. To avoid these two extreme cases, we appended a priori assumptions such as specifying the number of pixels in the structuring element in our conventional researches.

To avoid this drawback, we propose a novel method based on the combination of the minimum description length (MDL) principle [4] and morphological operations in this paper. The concept of MDL principle is that the most suitable model to encode a statistical ensemble is the simplest one in the sense that the description length of the combination of the result of encoding and the code dictionary is the shortest. The optimal model based on this concept reduces the data amount required for describing repetitive structures in the ensemble.

In this work, we describe a texture image by the combination of the shape of the primitive, the arrangement of the primitive, and the residue that is not expressed by the arrangement of the primitive. If we regard this description as a coding, the primitive corresponds to the code dictionary, and the arrangement to the result of encoding. Thus minimization of the total description length of the combination yields the optimal estimation of the primitive. Such inappropriate estimates as shown above are not selected by this optimization, since the description lengths of the primitive or the arrangement would be extremely large.

We have to note that the MDL principle intends to the minimization of description length for a statistical ensemble. If we regard the target image as a sample extracted from an ensemble, we cannot obtain enough information. However, if we restrict ourselves to texture images, the grains in a texture image are regarded as a sample set extracted from the population of variations of the primitive, and the minimization of the description length of the sample yields an estimate of the primitive.

We provide a method of the primitive estimation, and show experimental results of the ability of impulsive noise removal by morphological opening using the estimated primitive as the structuring element to show that the estimate is appropriate. Opening fits the structuring element into the objects in the target image and removes the residues that are too small. An impulsive noise is removed by opening since it is regarded as a set of small objects. If the structuring element is identical to the primitive of the texture, the quality of the target image is not degraded by the noise removal since the structuring element almost fits the grains. However, if the structuring element is far from the primitive, the structuring element cannot fit the grains, and the image is degraded by the removal of residues by the opening. It indicates that the quality of the estimate can be shown by the experiments of noise removal.

II. Definitions of morphological operations

We consider binary and gray scale images in this paper. In the case of binary images, opening of an object X in an image by a structuring element B has the following property:

$$X_B = \left\{B_z \mid B_z \subseteq X, z \in \mathbb{Z}^2\right\}, \tag{1}$$

where B_z denotes the translation of B by z, defined as follows:

$$B_z = \{b + z \mid b \in B\}. \tag{2}$$

Opening in the case of a gray scale object and a structuring element is defined using their umbrae. When the distribution of gray scale value is denoted by $f(x), x \in \mathbb{Z}^2$, The umbra $U[f(x)]$ is defined as follows:

$$U[f(x)] = \{(x,t) \in \mathbb{Z}^3 \mid -\infty < t \leq f(x)\}, \tag{3}$$

and the operation is applied to the umbrae.

The property of opening in Eq. (1) indicates that opening is regarded as the reconstruction of the object by an arrangement of the structuring element. This reconstruction removes regions smaller than the structuring element from binary objects or umbrae. It indicates that opening is a quantitative smoothing of images with respect to the structuring element.

III. MDL principle and texture modeling

A. Basic idea

Let A be an image object, and let B be a structuring element, as shown in Fig. 1. A small dot corresponds to a pixel. Opening A_B removes small regions, denoted by $\circ$. The remaining region, denoted by $\bullet$, can contain the structuring element inside, and the residual region cannot. In other words, the remaining region can be expressed by an arrangement of the structuring element. It indicates that the original image object is decomposed into the structuring element, its arrangement, and the residue.

If we restrict ourselves to texture images, the decomposition can be applied to each microstructure resembling each other, i. e. grain in the PGPC texture model. If the total description length of the structuring element, the arrangement, and the residue for all grains, the structuring element must resemble the grains by the following reasons: According to the above decomposition, the structuring element resembling the grains yields small residue. Although the smallest residue is achieved by the structuring element identical to the texture image itself, the description length of the structuring element is extremely large in this case. Although the smallest residue is also achieved by the one-pixel structuring element, the description length of the arrangement is extremely large in this case. The minimization of the total description length yields the structuring element taking the repetitive structure of the texture into account. This idea is the same as the concept of MDL, if the structuring element is regarded as a coding dictionary, and the arrangement and the residue are regarded as the results of encoding.

B. Primitive estimation procedure

The total description length L is defined as follows:

$$\begin{aligned} L = &\ (\text{description length of the structuring element}) \\ &+ (\text{description length of the arrangement} \\ &\quad \text{of the structuring element}) \\ &+ (\text{description length of the residue}). \end{aligned} \tag{4}$$

The information content of an event ω whose occurrence probability is $P(\omega)$ is defined as $\log(1/P(\omega))$ [*1]. The information content is employed as the definition of description length. Since we have no prior knowledge about the distribution of candidates of the structuring element, we assume that the distribution is uniform. Thus the description length of the structuring element is $\log Q_s$, where Q_s is the number of the candidates, and is regarded independent to the selection of structuring element and the minimization.

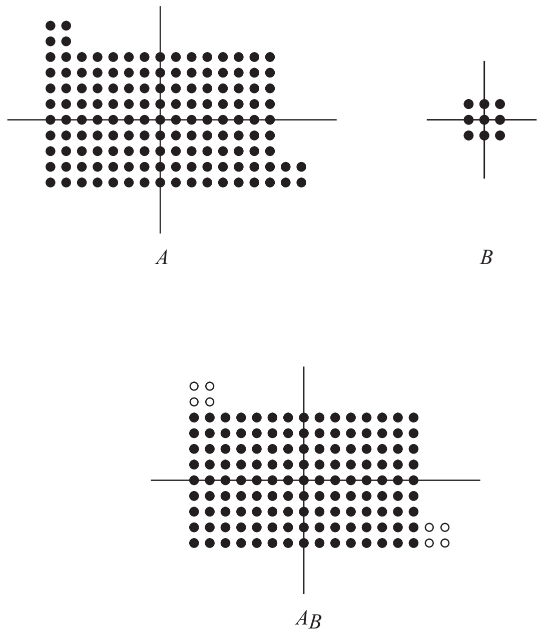
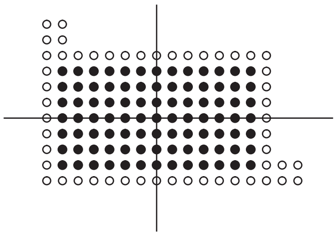
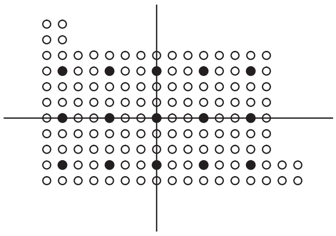


Fig. 1. Opening.

[*1] The base of the logarithm is 2 hereafter.



(a)



(b)

Fig. 2. Arrangement of the structuring element. (a) Result of erosion. (b) Reduced arrangement.

We have no prior knowledge about the distribution of the target texture images, either. If the number of pixels in the target image is Q_t, the appearance probability of a white pixel at each pixel position in a binary image is assumed to be $1/Q_t$. Thus the description length of the arrangement is $n_t \log Q_t$, where n_t is the number of pixel positions in the arrangement. Similarly, the description length of the residue is defined as $n_r \log Q_t$, where n_r is the number of pixels in the residue.

The number of pixel positions n_t in the arrangement does not directly relate to the number of pixels within the result of opening. The result of opening is reconstructed by locating the structuring element at each pixel in the result of erosion by the same structuring element. However, the arrangement at each pixel in the result of erosion is redundant. The reduced arrangement, as shown in Fig. 2 is obtained by removing one pixel if the reconstruction result is the same as the original, and repeating the removal one by one.

According to the above discussion, the total description length L' to be minimized is reduced as follows:

$$L' = n_t \log Q_t + n_r \log Q_t. \tag{5}$$

Since Q_t is independent to the minimization, the problem is reduced to the minimization of the number of pixels in the arrangement and the residue.

If we use only one structuring element, it does not always fit the grains well, and the residue can be large. We introduce a supplementary structuring element to decompose the residue in the same way. The description length with the supplementary structuring element is as follows:

$$L'' = n_t \log Q_t + n_{t'} \log Q_t + n_{r'} \log Q_t, \tag{6}$$

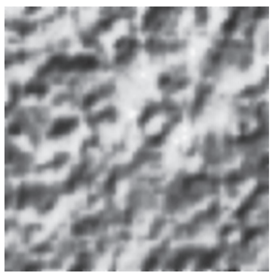
where $n_{t'}$ and $n_{r'}$ are the number of pixels in the arrangement of the supplementary structuring element and the number of pixels in the final residue, respectively.

The minimization is performed by using the simulated annealing. The procedure modifies the structuring element and measures the description length L''. If the description length is decreased by the modification, the modification is accepted. If it is increased, the modification is accepted at a small probability, and rejected otherwise. The procedure is repeated with decreasing the probability until the description length becomes sufficiently small.

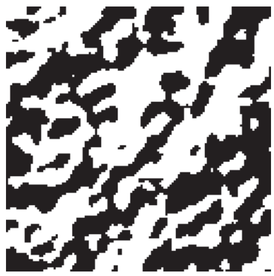
The primitive estimation of gray scale textures by a gray scale structuring element is performed by the following two steps.

1) The target texture is binarized by the adaptive binarization, whose threshold at each pixel is set to the mean of its surrounding pixels, as shown in Fig. 3 (b). The primitive is estimated as the binary structuring element by the minimization of the description length explained above.
2) The pixel values of the main structuring element estimated in the previous step are optimized. The sum of pixel values in the residue of opening is minimized in this step. The target texture image for the optimization is not the same as the original texture image, but the pixels which are black in the binarized image are replaced with zero, as shown in Fig. 3 (c). Since the fitting of structuring element based on opening is extraction of objects composed of brighter pixels than the background, the above replacement emphasizes the objects to be extracted.

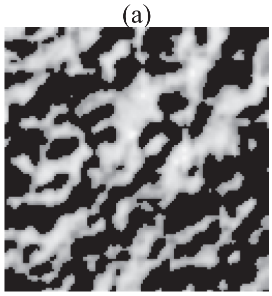
Note that the estimation by the minimization of the above description length cannot avoid the extreme structuring element that is the same as the target texture image itself,



(a)



(b)



(c)

Fig. 3. Binarization and extraction of brighter grains. (a) Original image. (b) Binarized image. (c) Extracted regions.

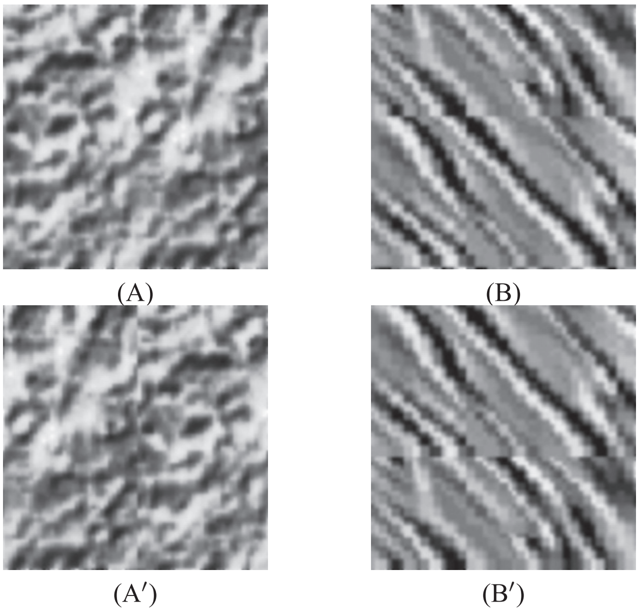


Fig. 4. Texture images used in the experiments.

since this extreme structuring element achieves the smallest arrangement and the smallest residue. However, it is not a practical problem since the above search of the optimal structuring element can hardly reach to the extreme one if the search starts from a sufficiently small structuring element.

IV. Experimental results

A. Results of estimation

Figure 4 shows the texture images used in the experiments. Each image contains 128×128 8-bit pixels. The pair of images A and A′ and the pair of images B and B′ are the pairs of different realizations of the same textures, respectively. Figure 5 shows the estimated primitives. One small square corresponds to a pixel, and the brightness of each square indicates its pixel value. The result shows that the estimated primitives for the different realizations of a texture are similar and visually resemble to the grains in the images. It indicates that the estimation is stable and reliable.

B. Noise removal by opening

If the primitive is properly estimated, the estimated primitive resembles the grains in the target texture image. It indicates that opening of the target image using the estimated primitive as the structuring element yields the output image that is quite similar to the original. Even if some pixels in the target image are randomly selected and replaced with white pixels, the result of opening still resembles the original. The white pixels are removed since they are regarded as small objects and removed by the opening. Thus the appropriateness of the estimated primitive in the previous subsection is shown by the experiment of the above noise removal in this subsection.

Figure 6 shows noisy images used in the experiment. Thirty percent of the pixels in each image of Fig. 4 are replaced with white pixels. Figure 7 shows the results of opening by the

TABLE I
Mean square errors between the results of noise removal and the originals.

	Estimate SE	Square SE 1	Square SE 2
Image A	142.96	382.71	266.44
Image B	79.42	226.55	290.80

estimated primitives. Figure 9 shows the results of opening by square structuring elements shown in Fig. 8. Table IV-B shows the mean square errors between the results and the original. The results show the appropriateness of the estimation.

V. Conclusions and future remarks

This paper has proposed an estimation method of primitives of textures based on mathematical morphology and the MDL principle. The method is simple and has an advantage over the conventional methods that unnatural restriction to the size of structuring element. We have assumed in this paper that the probability distributions of the selection of a structuring element and the appearance of a white pixel are uniform. If we have knowledge on their prior probability, we can introduce it and extend our idea. Moreover, the method requires the repetitive operation to reduce redundancy of the arrangement of primitive. A simpler algorithm to obtain the non-redundant arrangement is still a problem. The method estimates the primitive for grains brighter than the background. The estimation for grains darker than the background is possible by applying closing, and a combined method applicable to both of the cases is required. We are now working on them.

References

[1] M. Petrou, P. Garcia-Sevilla, *Image Processing: Dealing With Texture,* Wiley, 2006.

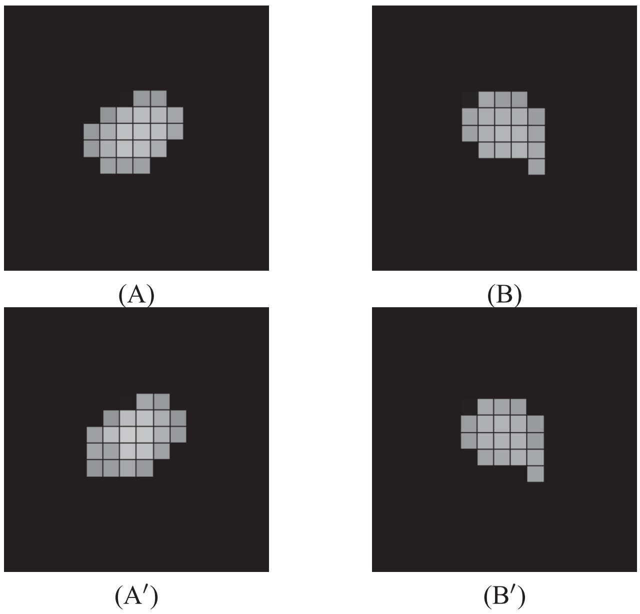
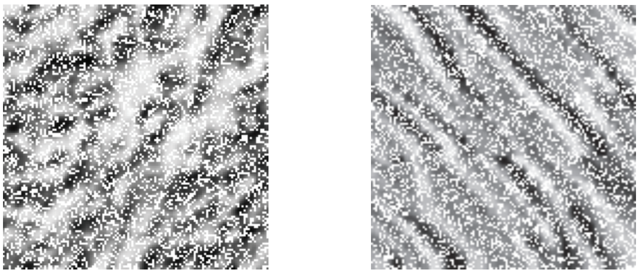
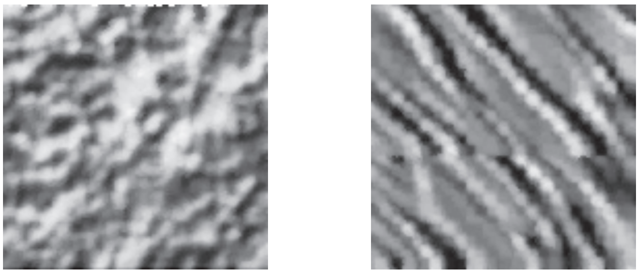


Fig. 5. Estimated primitives.



(A) (B)

Fig. 6. Noisy images.



(A) (B)

Fig. 7. Noise removal using the estimate primitives.

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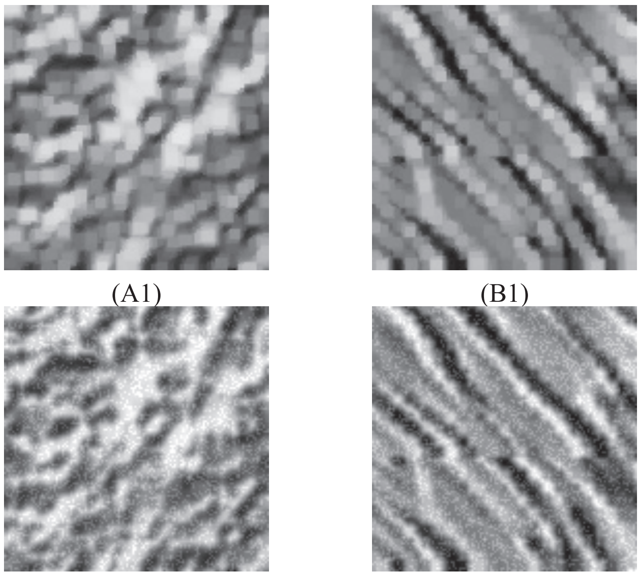
(1) (2)

Fig. 8. Square structuring elements for reference.



(A1) (B1)

(A2) (B2)

Fig. 9. Noise removal using square structuring elements (SEs). (A1) Image A by SE 1 in Fig. 8. (A2) Image A by SE 2. (B1) Image A by SE 1. (B2) Image B by SE 2.