Learning optimization of morphological filters with gray scale structuring elements

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Abstract. Mathematical morphology with gray scale structuring elements has attracted much attention, since combinations of the operations in this class can realize almost all noise-removing filters. However, the optimization method for the combination is still uncertain. In this paper, an optimization method for a mathematical morphological filter with gray scale structuring elements is proposed. This method is based on the concept of a neural network with morphological operations and on learning using simulated annealing. The method is also applied to gray scale bipolar morphological filters for image differentiation.

Subject terms: optical neural networks; mathematical morphology; nonlinear filters; image processing; simulated annealing; neural networks.

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1 Introduction

Filtering is one of the most fundamental techniques for image processing. Raw images should be properly filtered to prepare for high-level image processing such as pattern recognition. Recently, nonlinear filters have attracted more attention than linear ones, since they outperform the linear ones in such tasks as the improvement of nonlinearly corrupted images. The morphological filter is one of the most popular nonlinear filters. It has the significant characteristic that the image shapes and sizes are mainly treated, rather than pixel values themselves. This characteristic is useful for image recognition because image shapes are important for such high-level applications. It is also useful for removing impulsive noises, since such noises can be regarded as very small shapes in an image.

The basis of mathematical morphology[1] consists of two fundamental operations called erosion and dilation. Intuitively speaking, erosion shrinks image objects and dilation expands them, in the coordinate space of pixel positions. The structuring element determines how the objects are shrunk or expanded. Various combinations of the structuring elements realize many kinds of operators, which cover a very wide range of image operations. According to the filter theorem, all translation-invariant increasing filters can be expressed by morphological operations with gray scale structuring elements, followed by logical operations.[2]

For all their advantages, morphological filters are less widely used than linear ones. The reason is that an optimization method to design a morphological filter satisfying one's requirements is still unavailable. In the case of the linear filters, Fourier analysis and the criterion of the spatial frequency are sufficiently well established. A similar criterion for the morphological filters has not been discovered yet.

We recently proposed a novel approach to this problem with the binary morphological operations.[3] Our approach is based on the learning method of the layered neural network. We have constructed a layered neural network whose basic operation is the morphological operation. This method requires an example set of a noisy image and its ideal output. Its criterion for the design is minimizing the mean squared error between the filtering output of the noisy image and the ideal output.

The idea of this learning method has its foundation in the relationship between the rank-order filter and artificial neural networks.[4] We recently proposed a design method for the rank-order filter family using the formulation of layered neural networks.[5–7] Other researchers have investigated a similar method.[8] We utilized the close relationship between the rank-order filter family and the shift-invariant layered neural network. Each pixel is regarded as a neuron, and an image corresponds to a layer. The filtering operation to get the value of each neuron in the next layer is realized by shift-invariant interconnections to the neurons in the previous layer with certain weight coefficients and a nonlinear operation. The extent of the interconnections from each neuron corresponds to the extent of the filter window. In this formulation, the optimization methods for the conventional artificial neural networks can be directly applied under the restriction that the interconnections should be always shift-invariant.

In the case of the morphological filters, another learning method must be applied, since the mathematical morphological operation is not described analytically. We applied in Ref. 3 the simulated annealing method. The learning process is developed by setting or removing an interconnection between neurons at each position. The simulated annealing algorithm evaluates the variation of the error after the modification of the interconnections and determines whether the modification should be fixed or not.

In this paper, we propose an extension of our method proposed in Ref. 3 to handle filters with gray scale structuring elements. The class of translation-invariant increasing filters is proved by the filter theorem to contain almost all practical noise-removing filters. Thus we are motivated to extend the learning method for the optimization of the morphological filters to the case of gray scale morphological and logical operations. In this case the network is considered to have interconnections to which certain values are assigned. Simulated annealing was originally proposed to determine the on-off status—i.e., the existence—of the interconnections. We extend the algorithm to handle the modification of multivalued interconnections.

We also extend the method to networks based on the bipolar gray scale morphological operation. The bipolar morphological operation,[9] which contains bipolar erosion and bipolar dilation, is the fundamental operation of morphological image differentiation. The rank-order-based nonlinear differential operator (RONDO),[10] which is an extension of the rank-order filter to edge detection, is shown to be decomposable into bipolar morphological operations, as the median filter is decomposable into conventional morphological operations.[4] We previously proposed a learning method to optimize the binary bipolar morphological operations;[11] in this paper we show the application of our learning method to optimize bipolar morphological operations with gray scale structuring elements.

We describe in Sec. 2 the conventional mathematical morphology and the bipolar morphology with gray scale structuring elements. In Sec. 3 we present our network formulation based on the morphological operations. We also explain our learning algorithm based on simulated annealing. We show in Sec. 4 some experimental results of our method. We conclude our work in Sec. 5.

2 Mathematical Morphology with Gray Scale Structuring Elements

2.1 *Conventional Morphological Operations*

Mathematical morphology is an idea for modeling the processes of human recognition of visual information. Mathematical morphological operations are shift-invariant image manipulations and can be decomposed into two simple basic operations—dilation and erosion.

We first explain these basic operations in terms of binary structuring elements for binary images. These operations are defined as set operations. An image is assumed to be a set of pixel positions that constitute image objects. Let X denote a set representing an image. Let B be another set called a *structuring element.* The structuring element corresponds to the window of a filter. The Minkowsky set subtraction and addition are defined as follows:

$$\text{subtraction:} \quad X \ominus B = \{x | B_x \subseteq X\}; \tag{1}$$

$$\text{addition:} \quad X \oplus B = \{x | B_x \cap X \neq \emptyset\}, \tag{2}$$

where B_x denotes translation of B by x, defined as

$$B_x = \{z + x | z \in B\}. \tag{3}$$

The erosion and the dilation are defined as $X \ominus \check{B}$ and $X \oplus \check{B}$, respectively, where $\check{B}$ is defined as

$$\check{B} = \{-x | x \in B\}. \tag{4}$$

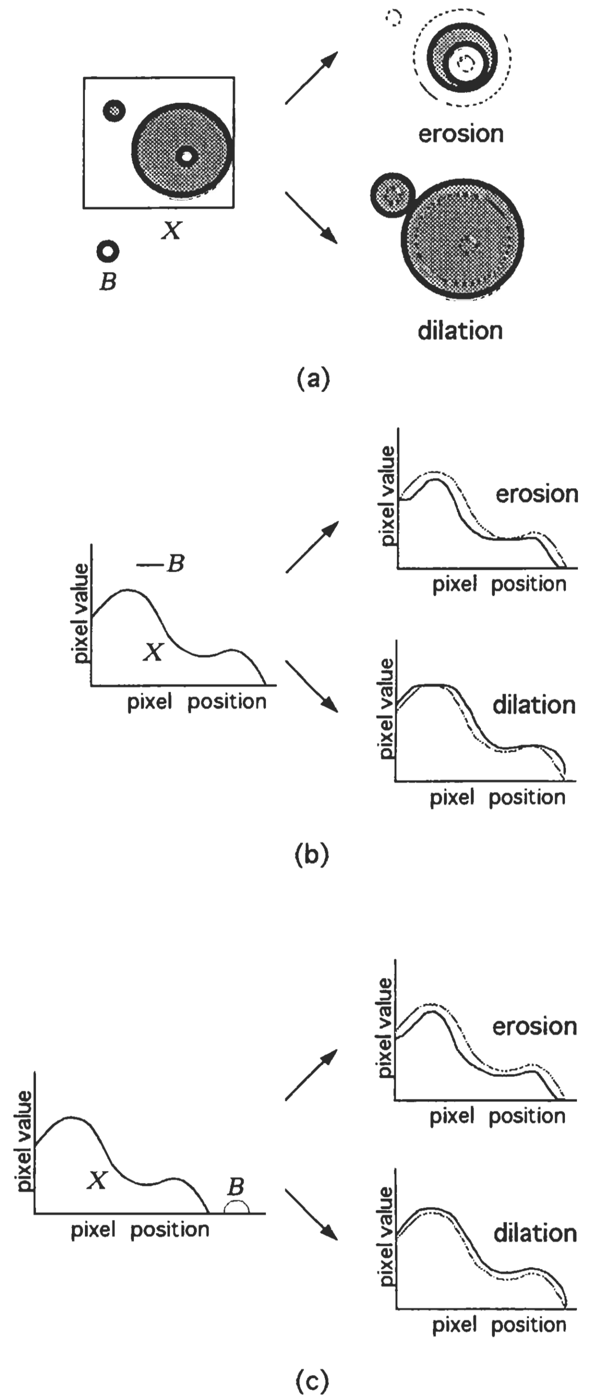

Fig. 1 Basic operations in mathematical morphology. (a) Binary operations. Image is shown by X, and structuring element by B. (b) Operations on gray scale image by binary structuring element. The one-dimensional case is shown for simplicity. The image X is shown by the solid curve, and the extent of the structuring element B by the horizontal line. The resultant images are shown by the solid curves over the original image shown by gray curve. (c) Operations on gray scale image by gray scale structuring element.

The effects of dilation and erosion are schematically illustrated in Fig. 1(a). Images and structuring elements, corresponding to the windows of the image filters, are treated in mathematical morphology. Suppose that the image

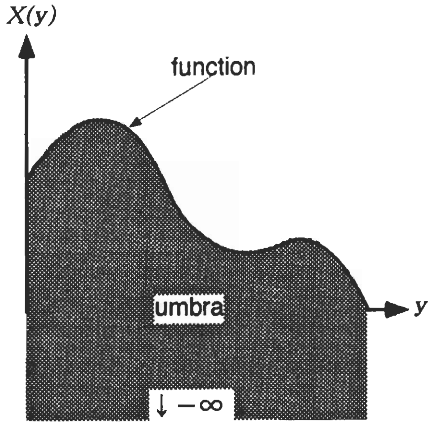

Fig. 2 Umbra of a function. The one-dimensional case is shown for simplicity. The umbra, shown by the shaded area, extends to $-\infty$ along the vertical axis.

moves along the boundary and through the interior of the structuring element. The dilation of the image is the whole area where the moving image sweeps at least once. The dilation expands the original image. The erosion of the image is the area that is included in the moving image all the time the image is moving. The erosion shrinks the original image.

The morphological operations for gray scale images can be defined by introducing the concept of *umbra*.[12] Consider a function $X(y)$, which expresses a gray scale image, where $y \in \mathrm{R}^n$, and consider an R^{n+1} coordinate system for $(y, X(y))$. In most cases, for image processing, $n = 2$. The umbra of a function is defined in this R^{n+1} coordinate system as the set of all coordinate points lower than the value of the function over the domain of the function, as shown in Fig. 2. Using this expression, the morphological operations of gray scale images are reduced to binary operations on the umbra of the function for the image. In the case that the structuring element is binary, the structuring element is defined as a range on the y axis. The mathematical operation is defined similarly to the case of binary images by moving the umbra of the function along the range of the y axis, as shown in Fig. 1(b). In this case, the erosion (dilation) is reduced to the set of the minima (maxima) of all the pixel values at each pixel position during the movement of the umbra.

The gray scale structuring elements are defined not by a range on the y axis but by another umbra. Dilation by gray scale structuring elements is defined by moving the umbra for the image along the umbra for the structuring element, i.e., not only along the y axis but also along the vertical axis. The erosion is defined as the complement of the dilation between the umbrae for the structuring element and the complement of the image. Here the complement of $X(y)$ is defined as $-X(y)$. These operations are illustrated in Fig. 1(c). They are also simply described by maximum/minimum and addition/subtraction operations.[12] Let a structuring element be $B(y)$. The erosion and dilation of $X(y)$ by $B(y)$ are defined as

$$\text{erosion:} \quad (X \ominus \check{B})(y) = \inf_{t \in A}[X(t) - B(t-y)], \tag{5}$$

$$\text{dilation:} \quad (X \oplus \check{B})(y) = \sup_{t \in A}[X(t) + B(t-y)], \tag{6}$$

where A is the extent of $B(y)$.

The importance of extending mathematical morphology to gray scale images and structuring elements lies in the existence of the filter theorem.[1] This theorem states that every increasing shift-invariant filter can be realized by erosions by a certain number of gray scale structuring elements followed by the pixelwise OR operation, or dilations followed by the AND operation. An *increasing* filter is defined as one that preserves the relations of inclusion between objects in images. If an increasing filter preserves an image feature, it preserves all larger features. If an increasing filter removes an image feature, it removes all smaller features. This property is quite natural for noise-removing filters. The shift-invariance property is also quite natural for such filters. Thus the filter theorem states that almost all noise-removing filters can be decomposed into morphological operations.

2.2 *Bipolar Morphological Operations*

The bipolar morphological operations are defined by combinations of the conventional morphological operations, as follows:

$$\text{bipolar erosion:} \quad \begin{cases} [X \ominus \langle B \rangle]^+ = (X \ominus B^+) \backslash (X^c \ominus B^-), \\ [X \ominus \langle B \rangle]^- = (X^c \ominus B^+) \backslash (X \ominus B^-), \end{cases} \tag{7}$$

$$\text{bipolar dilation:} \quad \begin{cases} [X \oplus \langle B \rangle]^+ = (X \oplus B^+) \backslash (X^c \oplus B^-), \\ [X \oplus \langle B \rangle]^- = (X^c \oplus B^+) \backslash (X \oplus B^-), \end{cases} \tag{8}$$

where X^c is the set complement of X. Bipolar morphological operations require a pair of sets as the structuring elements. The structuring element B^+ is called the positive element, and B^- is called the negative element. A pair of elements is required because calculating the image differentiation needs at least two pixels. We use here the symbol \ to denote a fuzzy-set operation called bounded difference, defined as follows:

$$\mu_{X \backslash Y}(x) = \max(\mu_X(x) + \mu_Y(x) - 1, 0), \tag{9}$$

where $\mu_X(x)$ means the membership function of an element x of the fuzzy set X. In the case of crisp sets, i.e., binary images, the bounded difference is reduced to simple set intersection. Bipolar morphological operations map an input image set to two resultant sets $[\,]^+$ and $[\,]^-$, because an image-differentiating operation outputs positive and negative values in general. The set $[\,]^+$ corresponds to a positive differential value (i.e., edge image), and $[\,]^-$ to a negative. This definition is still valid for the case of gray scale structuring element if erosion and dilation are defined for grayscale structuring elements.

Bipolar erosion outputs only complete edges that are exactly along the orientation of the structuring element and whose length is greater than or equal to the size of structuring elements. Bipolar erosion can be thought of as "the

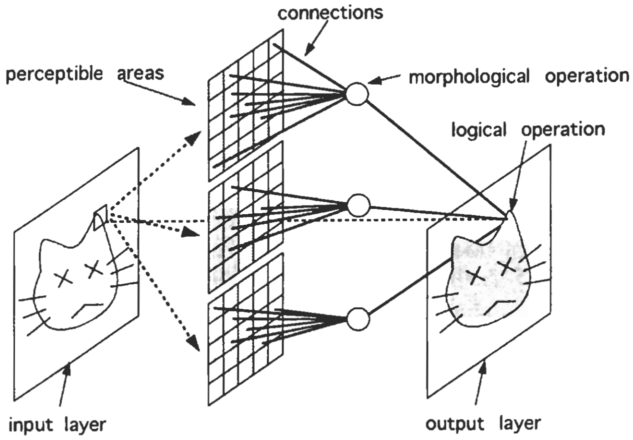

Fig. 3 Network architecture based on morphological operations.

strictest edge estimation.'' Bipolar dilation detects any pixels that potentially belong to an edge. It can be thought of as ''the most tolerant edge estimation.'' Many morphological edge detectors can be constructed by combining the bipolar morphological operations with several structuring elements by logical operations. For example, RONDO is constructed from OR of the bipolar erosions or from AND of the bipolar dilations.

3 Network Configurations and Learning Algorithm

3.1 *Network Configurations*

Our novel neural network is based on mathematical morphological operations with multiple structuring elements and on multivalued logical operations. The principle of our network formulation is schematically illustrated in Fig. 3. The basic network has two layers, the input layer and the output one. Each layer corresponds to an image, and each neuron corresponds to a pixel. A neuron in the output layer is connected to several overlapping sets (called *perceptible areas*) of neurons that lie in the input layer around the corresponding position of the output neuron. Each perceptible area has different patterns of connections between a neuron in the output layer and some neurons in each perceptible area. Because of the shift invariance of the morphological operation, the connecting pattern for every output neuron is the same. The operation to get the values of the neurons in the output layer is as follows: Each neuron in the output layer contains several perceptible areas. In each perceptible area, the connected neurons are regarded as constructing a structuring element. Then a morphological operation is carried out, and the neuron determines its value by a multivalued logical operation on the results of the morphological operation for each perceptible area.

In the conventional neural network, the output is determined by some thresholdlike operations on the summation of the values of the connected neurons in the input, so that it is suitable for handling binary values. Thus handling gray scale images requires some conversion to binary images, such as threshold decomposition. On the contrary, all the operations in the proposed network are calculated in gray scale, and no binarizing techniques are needed. For this reason the morphological network is natural for the design of gray scale image processing.

This architecture can be extended to multilayered networks with the use of *opening* (which is defined as erosion followed by dilation) or *closing* (dilation followed by erosion), with multiple structuring elements. This multilayered architecture, shown in Fig. 4, has an intermediate layer that corresponds to each perceptible area. Each intermediate

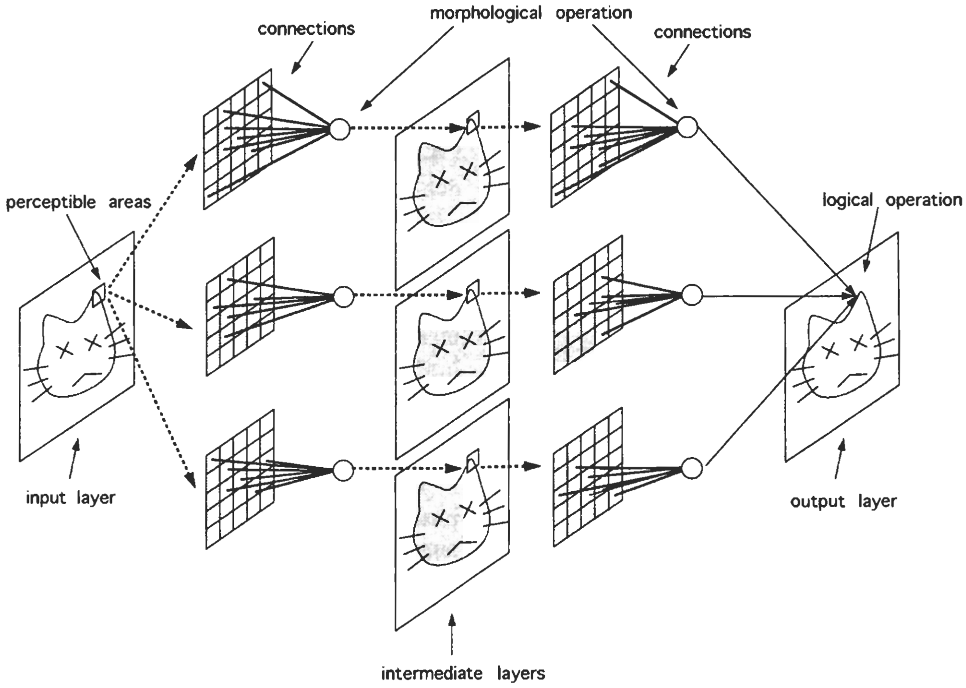

Fig. 4 Multilayered network architecture for opening and closing.

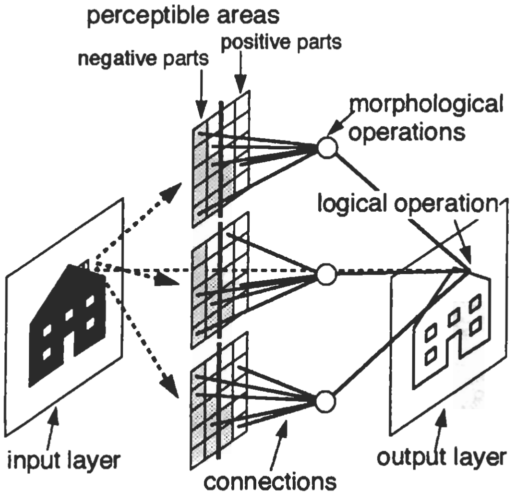

Fig. 5 Network architecture for bipolar morphological operations.

layer is the output of the basic network with the corresponding perceptible area as input. Since each basic network here has only one perceptible area, the basic network requires no logical operation. These basic networks are based on erosion (dilation) in the case of opening (closing). Then another set of the basic networks is applied for the set of the intermediate layers. Each perceptible area assigned to a given intermediate layer is the reverse of the corresponding perceptible area in the first half in this cascade, because of the relation to opening or closing. These basic networks in the second half are based on dilation (erosion) in the case of opening (closing). The values in the output layer are then calculated by a logical operation from the outputs of the basic networks. This operation is OR (AND) in the case of the opening (closing).

The network architecture for the morphology of gray scale images with binary structuring elements is easily extended to operations by gray scale structuring elements and to bipolar morphological operations. In the case of the network, to optimize the operation by gray scale structuring elements, a gray scale value is assigned to each connection. This value is used as the gray scale value of the structuring elements for the gray scale morphological operation in each perceptible area. In the case of the optimization of the bipolar morphological operations, there is no difference except that each perceptible area has positive and negative parts, and the network performs the bipolar morphological operations, as shown in Fig. 5.

3.2 *Learning Algorithm*

The learning algorithm optimizes the connection patterns (i.e., the shapes of the structuring elements) and also the gray scale value assigned to each connection in the perceptible areas. This algorithm requires an example of a corrupted image and its ideal output. The object of the learning algorithm is to minimize the error between the output of the network from the corrupted image and the ideal output. The minimization is gradually achieved during iterations. At an iteration, the algorithm randomly alters one connection of one perceptible area. We apply two kinds of alterations: *binary alteration* and *gray scale alteration.* The binary alteration is the same as the method we proposed before for the optimization of binary morphological operations. This operation sets or removes one connection. If a connection is established, the value assigned to this connection is supposed to be zero. The gray scale alteration varies the values assigned to the connection. More precisely, the procedure is described as follows:

Generate unity or zero randomly in equal probability:
If unity is generated,
  If this connection is not set, set the connection and assign zero.
  Else 1 is added to the value assigned to this connection.
If zero is generated,
  If this connection is set and the value is greater than 0, then 1 is subtracted from the value.
  Else this connection is set and the value is 0; the connection is removed.
  Else the connection suffers no effect (is still not set).

Then the learning algorithm estimates the errors between the outputs of the actual network and the ideal one for both before and after the alteration. It calculates the difference of these two errors. Then it determines whether to fix this alteration or not, following the principle of simulated annealing. Simulated annealing is a minimization mechanism that makes the error reach the global minimum with high probability, as follows: The alteration is fixed whenever it causes reduction of the error. If the alteration causes an increase of the error, it is fixed with some probability smaller than 0.5. Formally, let the difference of the error be x, and let the probability of fixing the alteration be $f(x)$. The relationship between x and $f(x)$ is as follows:

$$f(x)=\frac{1}{1+\exp(x/T_n)}, \tag{10}$$

where T_n is called the temperature parameter at the n'th iteration. This parameter decreases with the progress of the iteration. From this equation we get that larger error causes smaller probability, and that progress of the iteration also makes the probability smaller. The learning algorithm iterates these alterations and determinations until the connections do not vary any further.

In the practical cases of the optimization of filters for image processing, some restrictions on the shapes of the window or the perceptible area are often required. Since mathematical morphology is a well-adapted framework for image manipulation, it is easy to attach additional restrictions to the learning procedure.

4 Experimental Results

In all the experiments in this section, every image, unless otherwise specified, contains 100× 100 pixels, and each pixel has an 8-bit gray scale value.

4.1 *Noise Removal by Morphological Filters with Gray Scale Structuring Elements*

In all the experiments in this subsection, we optimized the open-closing filter with three structuring elements. The extent of each structuring element is 5×5 pixels.

Fig. 6 Original image for the experiment on noise removal.

Fig. 9 Output of the median filter.

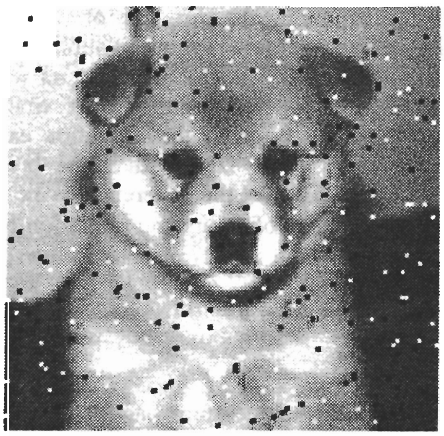

Fig. 7 Noisy image, corrupted by salt-and-pepper noise.

Fig. 8 Output of the optimized filter.

Figure 6 shows the original image for the learning procedure. The filters are optimized in the sense that the outputs from the sample noisy image corrupted by certain noises resemble the original image as well as possible. Every learning procedure applies binary alteration at first, and then applies gray scale alteration. The learning procedure was iterated 3000 times, with $T_0 = 5{,}000{,}000$ and $T_n = 0.995T_{n-1}$.

Figure 7 shows the noisy image corrupted by salt-and-pepper noise of probability 3%. Figures 8 and 9 show the outputs of the optimized filter and the median filter with a 3×3 window, respectively. This result indicates that the optimized filter preserves the image details better. Figure 10 shows the resultant structuring elements. The blank cells are outside the structuring elements. Table 1 shows the comparison of the mean squared errors per pixel between the original image and the noisy image, as well as the output of the median filter with 3×3 window, the optimized open-closing filter with three binary structuring elements whose extent is 5×5 pixels, and the optimized filter with gray scale structuring elements. This comparison indicates that introducing gray scale structuring elements achieves better optimization than using binary structuring elements.

Figure 11 shows another image corrupted by the same type of noise as the image for the optimization procedure. Figures 12 and 13 show the output of the optimized filter and that of the median filter with 3×3 window, respectively. It is shown that the optimized filter removes noise effectively while preserving the details better than the conventional median filter. Note that the noisy pixels near the boundary of Fig. 12 are not related to the performance of the optimized filter: they remain unfiltered, since the opti-

Fig. 10 Resultant structuring elements.

Table 1 Comparison of the filters for noise removal.

Image	Error
Noisy image	606.3812
Median filter	26.9451
Optimized filter with binary structuring elements	20.9111
Optimized filter with grayscale structuring elements	10.2102

mized filter has structuring elements of 5×5 pixels, so that the pixels whose distance from the boundary is less than 3 pixels cannot be filtered.

4.2 *Edge Detection in a Corrupted Image by Gray Scale Bipolar Morphological Operators*

In all the experiments in this subsection, we optimized the bipolar dilation-AND operator with seven structuring elements. The bipolar dilation-AND operator outputs the minimum of the absolute value of the bipolar dilation by each structuring element. The extent of each structuring element is (vertical 3)× (horizontal 1) for both positive and negative parts. Figure 14 shows the original image for the learning procedure. The operators are optimized to extract the edge image, which is as close as possible to the edge image extracted from the original noise-free image (i.e., the ideal edge image). Every learning procedure applies binary alteration at first, and then applies gray scale alteration. The learning procedure is iterated until T_n reaches 0.01 or smaller. Here $T_0 =$ 10,000,000,000, and T_n decreases exponentially.

Figure 15 shows the noisy image corrupted by salt-and-pepper noise of probability 1%. Figures 16 and 17 show the outputs of the optimized operator with gray scale structuring elements and the optimized filter with binary structuring elements, respectively. Figure 18 shows the resultant structuring elements of the optimized filter with gray scale structuring elements. The thick vertical line divides a structuring element into the positive part on the right side and

Fig. 12 Output of the optimized filter for the corrupted image in Fig. 11.

Fig. 13 Output of the median filter.

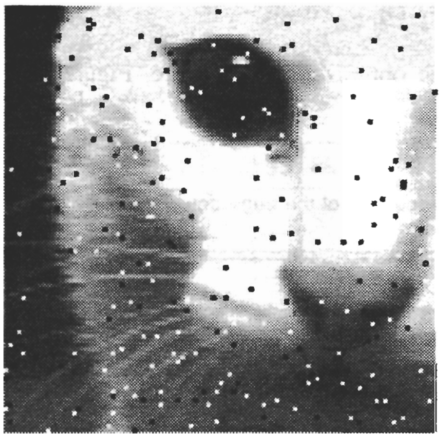

Fig. 11 Another corrupted image.

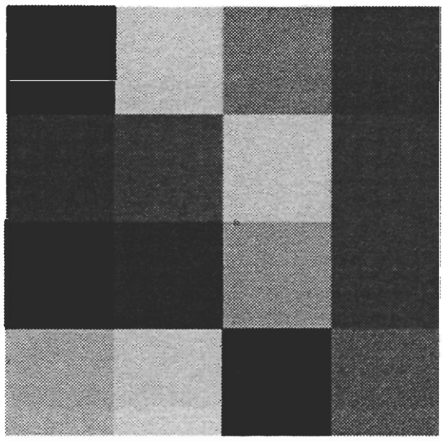

Fig. 14 Original image for the experiment on edge detection.

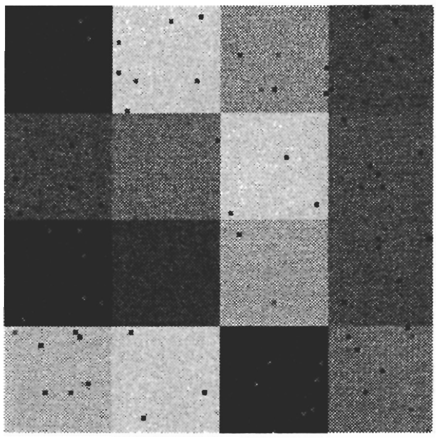

Fig. 15 Noisy image, corrupted by salt-and-pepper noise.

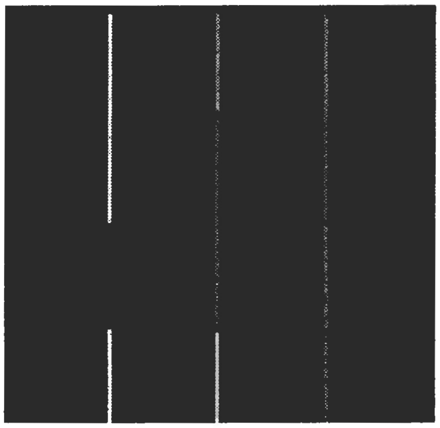

Fig. 16 Output of the optimized operator with gray scale structuring elements.

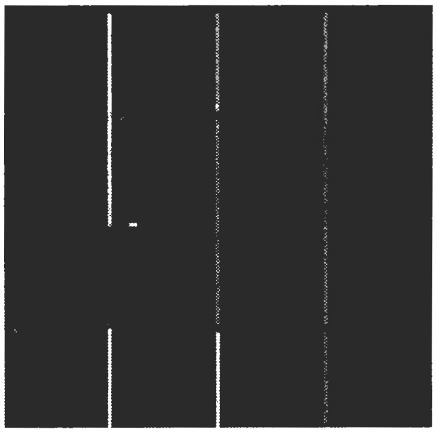

Fig. 17 Output of the optimized operator with binary structuring elements.

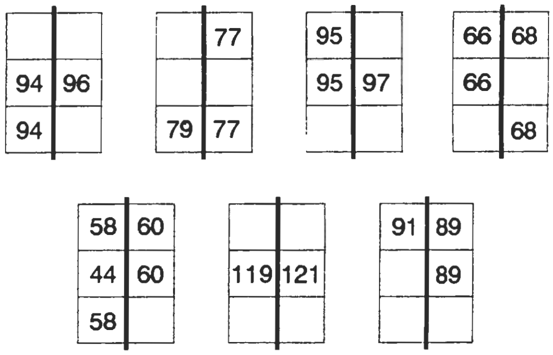

Fig. 18 Resultant structuring elements.

the negative part on the left side. The blank cells are outside the structuring elements. Table 2 shows a comparison of the mean squared error per pixel between the detected edge images and the ideal edge image for the optimized morphological filter with gray scale structuring elements and for that with binary ones. This comparison indicates that introducing gray scale structuring elements achieves better optimization than using binary structuring elements. The optimized filter with gray scale structuring elements outperforms that with binary elements, which we proposed before.[11]

Figure 19 shows the noisy image corrupted by the Gaussian noise of standard deviation 10. Figures 20 and 21 show, respectively, the output of the optimized operator with gray scale structuring elements and that of a linear edge detector with the same size of window as the structuring element of the optimized operator and whose coefficients are all 1, respectively. Figure 22 shows the optimized structuring elements. Table 3 shows a comparison of the mean squared errors per pixel between the filter outputs and the original image for the optimized morphological filter with gray scale structuring elements and for the linear edge detector. These results indicate that the bipolar morphological filter has better performance on linearly corrupted images than the linear operator, which is known to be suitable for this situation.

To verify how well the operator is generalized, the optimized filter was applied to an image different from those used for the optimization procedure. Figure 23 shows the

Table 2 Comparison of the edge detectors in the case of salt-and-pepper noise.

Image	Error
Noisy image	930.0626
Optimized operator with binary structuring elements	5.3381
Optimized operator with grayscale structuring elements	1.9261

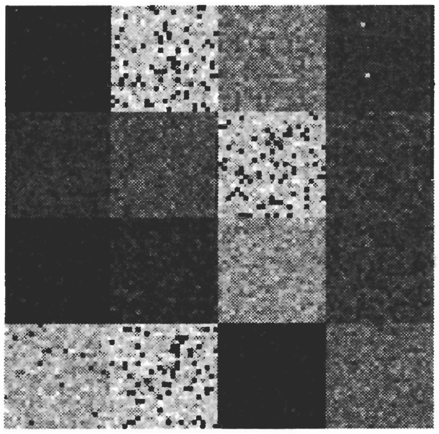

Fig. 19 Noisy image, corrupted by Gaussian noise.

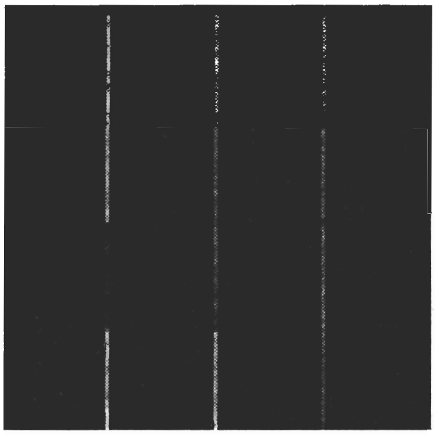

Fig. 20 Output of the optimized operator.

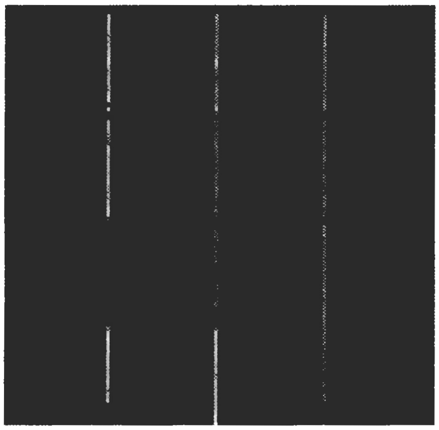

Fig. 21 Output of the linear edge detector.

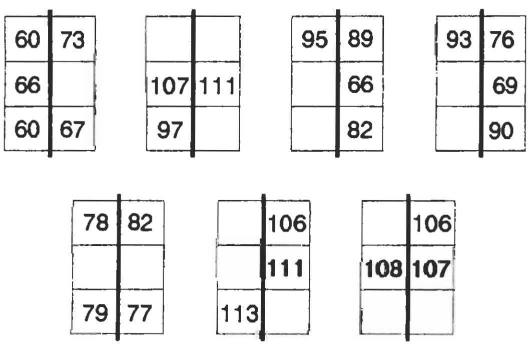

Fig. 22 Resultant structuring elements.

Table 3 Comparison of the edge detectors in the case of Gaussian noise.

Image	Error
Noisy image	72.6444
Linear edge detector	36.1249
Optimized operator with grayscale structuring elements	14.9406

noiseless test image, and Fig. 24 shows the ideal edge image for reference. In this experiment, each of these images contains 256×240 pixels, the operators are applied both vertically and horizontally, and the outputs are combined by taking the maximum. Figure 25 shows a noisy image corrupted by the same type of noise as Fig. 15. The resultant image output by the filter optimized for Fig. 15 is shown by Fig. 26. This result indicates that the optimized filter is well generalized to extract edges of any direction in the noisy image.

5 Conclusions

We have proposed in this paper a novel optimization method for a mathematical morphological filter with gray scale structuring elements. This method is based on the concept of the neural network with morphological operations and learning using simulated annealing. We have also applied gray scale bipolar morphological filters for image differentiation. We have shown experimentally that the optimized morphological filters and edge-detecting operators with gray scale structuring elements have excellent performance for images with various types of noise. These results also indicate that the morphological filters and operators with gray scale structuring elements have excellent capability for noise removal in various types of image processing.

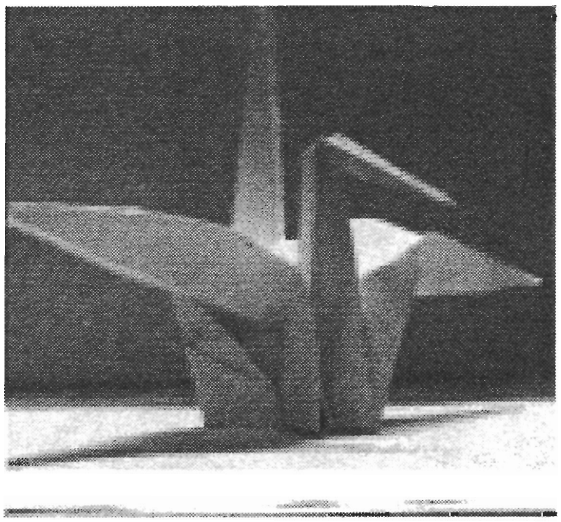

Fig. 23 Test image for edge detection.

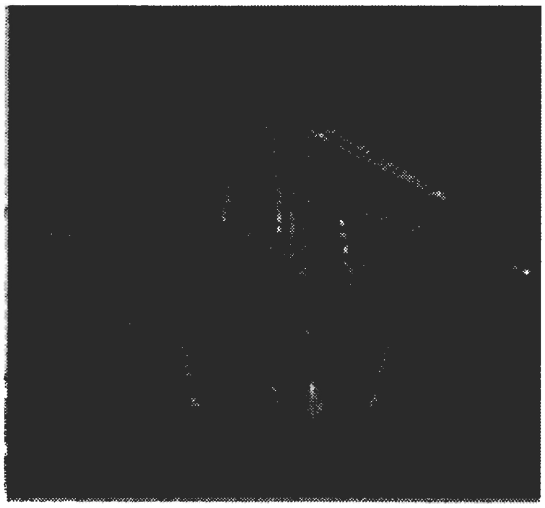

Fig. 24 Ideal edge image.

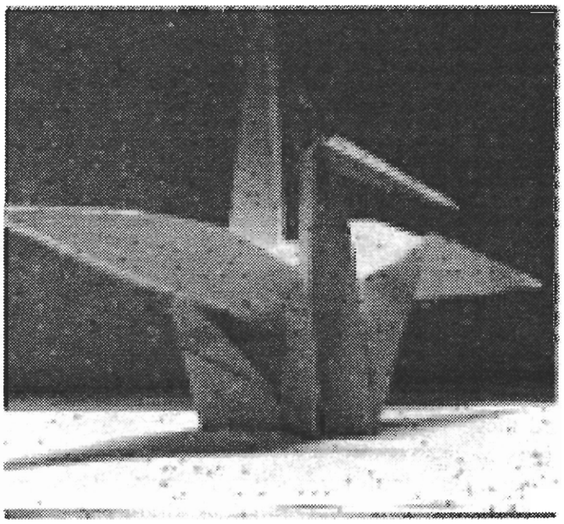

Fig. 25 Noisy test image.

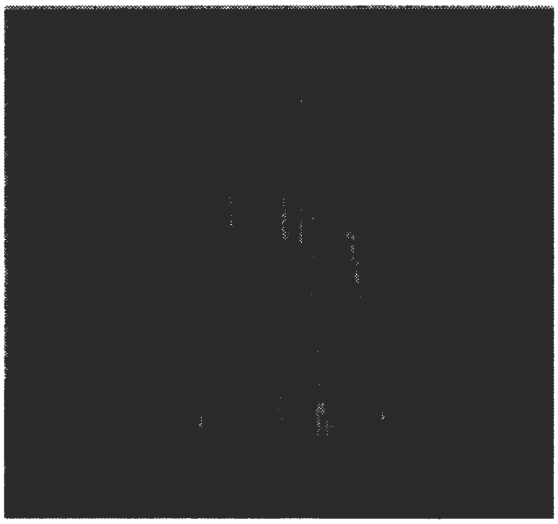

Fig. 26 Output of the optimized operator with gray scale structuring elements.

It will be a future project to introduce *a priori* knowledge about shapes of structuring elements in order to make the learning procedure more efficient.

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