PAPER

Morphological Multiresolution Pattern Spectrum

Akira ASANO†, *Member and* **Shunsuke YOKOZEKI†**, *Nonmember*

SUMMARY The pattern spectrum has been proposed to represent morphological size distribution of an image. However, the conventional pattern spectrum cannot extract approximate shape information from image objects spotted by noisy pixels since this is based only on opening. In this paper, a novel definition of the pattern spectrum, morphological multiresolution pattern spectrum (MPS), involving both opening and closing is proposed. MPS is capable of distinguishing details from approximate information of the image.
key words: *image analysis, multiscale analysis, mathematical morphology, pattern spectrum*

1. Introduction

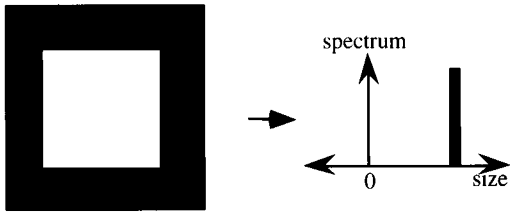
Recently, quantitive characterization of shapes of figures in digital images has been investigated. The mathematical morphology [1] is a key theory in this field. It introduced the structuring element (SE), a small figure similar to the window of image filters, and an image operation called opening. The opening by an SE removes fragments smaller than the SE and preserves the rest of the figures. The significance of the opening is to eliminate smaller fragments according to the size of the SE, i.e. a quantitative measure.

The pattern spectrum (PS) [2] was proposed as a method to extract contribution to each size from an image using opening. The opening removes fragments whose sizes are smaller than the size of SE. If the SE is gradually magnified while preserving the shape and the opening is performed by each magnified SE, we get a sequence of the resultant images by the openings with the SEs of increasing sizes. The n-times magnification is generated by n times of recursive dilations of the SE itself. The value n is referred to "size" here. The fragments smaller than the corresponding SE have been removed from each corresponding image in the sequence. The area of differences between the resultant images for sizes n and $n+1$ indicates the contribution of the figures in the original image to size n. This is defined as the spectral value of PS for size n.

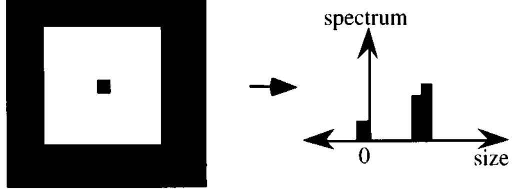
PS is similar to the Fourier spectrum in the field of linear signal processing. While the Fourier transformation decomposes an image to the contributions to the spatial frequencies, PS decomposes an image to the contribution to sizes. In other words, PS expresses how many objects of a certain size which are similar in shape to an SE are contained in an image.

Manuscript received January 16, 1997.
† The authors are with the Department of Mechanical System Engineering, Kyushu Institute of Technology, Iizuka-shi, 820 Japan.

The conventional PS is useful to quantitatively handle image decomposition [3], image classification [4], [5], and skeltonization [2]. This method is, however, based only on openings, and it causes a defect. Compare a square and the same square with a small black spot, shown in Fig. 1 (white figure on black background). By the conventional PS with a square SE, the complete square itself is similar to the SE so that only one peak appears in the spectrum, as shown in Fig. 1(a). However, in the case of the rectangle with a spot, shown in Fig. 1(b), the openings by gradually magnified SEs never remove the spot. Thus the whole figure is no longer regarded as one large rectangle and the peak found in the spectrum in Fig. 1(a) does not appear in the spectrum Fig. 1(b), though the shape of the rectangle in Fig. 1(b) is quite close to that in Fig.1(a). This suggests that a small modification in an image causes quite different spectra, and that the information about approximate view of the figure is no longer extracted by the conventional PS in such cases. This result is quite different from human impression of the images. This example also suggests that the conventional PS



(a)



(b)

Fig. 1 A defect of the conventional PS. (a) spectrum for a complete square (white on black background). (b) spectrum for a square with a spot of one pixel. The peak for the large spot vanishes, and the spetrum is quite different from (a).

cannot separate noisy spots of small sizes from approximate views of large objects.

This paper presents a novel concept "morphological multiresolution pattern spectrum (MPS)" which separates noisy spots of small areas from approximate information of larger figures. MPS introduces the morphological closing operation to the decomposition process of the conventional PS. The closing is the complement of opening: it fills up spots smaller than SE in objects in images. Just before applying the opening by the SE of size n to calculate the contribution to size n, the closing with the same SE is applied to fill up spots of size n. The clos-opened image generated by this process of size n is used for the process of size $n+1$, i.e. MPS is calculated recursively. On the step when the contribution to a size is calculated, black spots smaller than the size have been removed by the preceding closings. Thus MPS can extract approximate views of figures, and can separate smaller spots from larger figures.

The outline of the mathematical morphological operations and the definition of PS and MPS are explained in Sect. 2. In Sect. 3, examples on a set of typical figures and on a real image are shown. The concluding and future remarks are explained in Sect. 4.

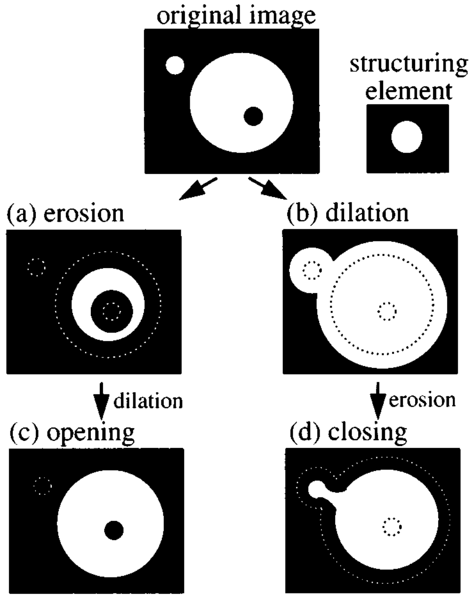


Fig. 2 Basic morphological operations. (a) erosion, (b) dilation, (c) opening, and (d) closing.

2. Morphological Multiresolution Pattern Spectrum

A. Mathematical Morphological Operations

We first show some basic and important morphological operations [6]. We assume binary images for discussions in this paper. Images are regarded as sets whose elements are the coordinates of the pixels contained by the objects in the image. Here the structuring element is introduced: this is also a set of coordinates and is compared to the window of image processing filters. The most basic morphological operations, erosion and dilation, of image X by the structuring element B are defined as follows:

$$\begin{cases} \textit{erosion}: X \ominus \check{B}, \\ \textit{dilation}: X \oplus \check{B}, \end{cases} \tag{1}$$

where $\check{B}$ denotes the inversion of B against the origin, defined as follows:

$$\check{B}=\{-b|b\in B\}, \tag{2}$$

and, $\ominus$ and $\oplus$ are called the Minkowski set subtraction and addition, respectively, defined as follows:

$$\begin{cases} X \ominus B=\bigcap_{b\in B} X_b, \\ X \oplus B=\bigcup_{b\in B} X_b, \end{cases} \tag{3}$$

where X_b denotes the translation of X by the vector $\boldsymbol{b}$, defined as follows:

$$X_b=\{x+b|x\in X\}. \tag{4}$$

Applied to an image, the erosion shrinks the objects and eliminates fragments smaller than the SE, and the dilation expands the objects by the size of the SE and fills up black spots smaller than the SE. Here the term "smaller than the SE" means "completely included by the SE."

The opening and closing are combinations of the erosion and dilation, and are the most important basic operations, defined as follows:

$$\begin{cases} \textit{opening}: X_B=\left(X \ominus \check{B}\right) \oplus B \\ \textit{closing}: X^B=\left(X \oplus \check{B}\right) \ominus B \end{cases}. \tag{5}$$

Figure 2 illustrates the effect of these four operations. The opening shrinks objects and eliminates portion of objects smaller than SE by the erosion, and then the following dilation restores shrunk objects. The portion larger than the SE is not completely eliminated by the preceding erosion and then restored by the following dilation. However, the portion smaller than SE is completely eliminated by the erosion and never restored by the dilation. Thus the opening eliminates portion smaller than SE while preserving other portion of the objects. The closing is the complement: it fills up smaller spots while preserving the other portion of the background. This ability of opening and closing to quantify the size distribution of objects are important for defining PS and MPS.

B. Pattern spectrum and multiresolution pattern spectrum
The pattern spectrum (PS) of size n by an SE is defined as the area of white pixels which are contained by the images opened by the similarly magnified sets of SEs of both size n and size $n+1$. Let nB be the similar magnification of a structuring element B of size n, defined as follows:

$$nB=\underbrace{B\oplus B\oplus\ldots\oplus B}_{n\text{-1 times of }\oplus}\quad n>0,\qquad 0B=\{\mathbf{0}\}. \tag{6}$$

Then the PS of image X by a structuring element B for size n, denoted as PS(X, B, n) is defined as follows:

$$\mathrm{PS}(X,B,n)=A(X_{nB}-X_{(n+1)B}) \tag{7}$$

where A(Y) is the cardinality of the set Y (i.e. area of objects in Y, or the number of white pixels in the image Y), and X-Y denotes the set difference, defined as follows:

$$X-Y=\{a|a\in X \text{ and } a\notin Y\}. \tag{8}$$

Since the opening operation removes the portion smaller than the SE, the difference of images opened by the SEs of size n and $n+1$ contains the portion whose size is exactly n.

The multiresolution pattern spectrum (MPS) is defined by introducing the closing processes into the definition of the conventional pattern spectrum. At first we define MPS for $n\geq0$. MPS of size 0 by the structuring element B is defined as follows:

$$\mathrm{MPS}(X,B,0)=A(X-X_B) \tag{9}$$

i.e. the same as PS(X, B, 0). The MPS for size $n>0$ is defined as follows:

$$\mathrm{MPS}(X,B,n)=A((\mathrm{OC}(X,B,n))_{nB}-(\mathrm{OC}(X,B,n))_{(n+1)B}) \tag{10}$$

where OC(X, B, n) is defined recursively, as follows:

$$\mathrm{OC}(X,B,n)=((\mathrm{OC}(X,B,n\text{-}1))_{(n-1)B})^{nB}\quad n>1,\qquad \mathrm{OC}(X,B,1)=X^B,\qquad \mathrm{OC}(X,B,0)=X. \tag{11}$$

PS and MPS for $n\leq0$ are defined by interchanging the opening and the closing in the above definitions for size $|n|$. Note that there are two definitions for size n=0: one using the definition for $n\geq0$ is denoted MPS(X, B, +0) and the other, using the definition for $n\leq0$, is denoted MPS(X, B, -0). These two are different: MPS(X, B, +0) is the area eliminated by the opening of size 1, and MPS(X, B, -0) is the area filled by the closing of size 1.

When MPS of size $n\geq0$ is calculated, a black spot smaller than size n within an object has been filled up by the closing operation of size n, and no longer exist in the image OC(X, B, n). Thus the portion of size n is extracted by opening operations without any influences of noisy spots smaller than n. Note that MPS is obviously not anti-extensive.

The image $(\mathrm{OC}(X,B,n))_{nB}$, which is the cumulatively clos-opened image by the SEs of size 0, 1, 2, ..., n, indicates approximate views of the original images which do not contain fragments smaller than the SE of size n. The size n is regarded as the degree of approximation, and as a kind of generalization of the term "resolution." The process of the calculation of MPS is extraction of the spectra from the images of reduced "generalized resolution." This is the reason to call MPS "morphological multiresolution."

3. Examples

Firstly we show examples of PS and MPS for some typical figures. In this example SE of size 1 is 3×3-pixel square, and spectra for only positive sizes are calculated. Figure 3(a) shows a complete square of 13×13 pixels. The spectra by PS and MPS are the same in this case, as shown in Table 1(a): A

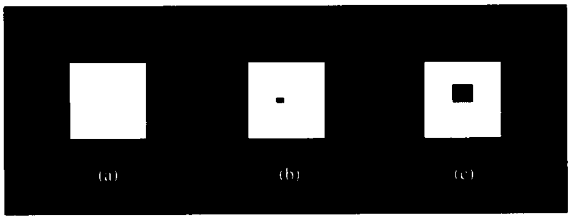


Fig. 3 Examples of typical figures. (a) square of 13×13 pixels. (b) square with one-pixel spot. (c) square with 3×3-pixel spot.

Table 1 Spectral values by PS and MPS.

(a) complete square

size	0	1	2	3	4	5	6	7
PS	0	0	0	0	0	0	169	0
MPS	0	0	0	0	0	0	169	0

(b) square with a spot of one pixel

size	0	1	2	3	4	5	6	7
PS	0	0	77	91	0	0	0	0
MPS	0	0	0	0	0	0	169	0

(c) square with a spot of 3x3 pixels

size	0	1	2	3	4	5	6	7
PS	0	12	148	0	0	0	0	0
MPS	0	12	0	0	0	0	169	0

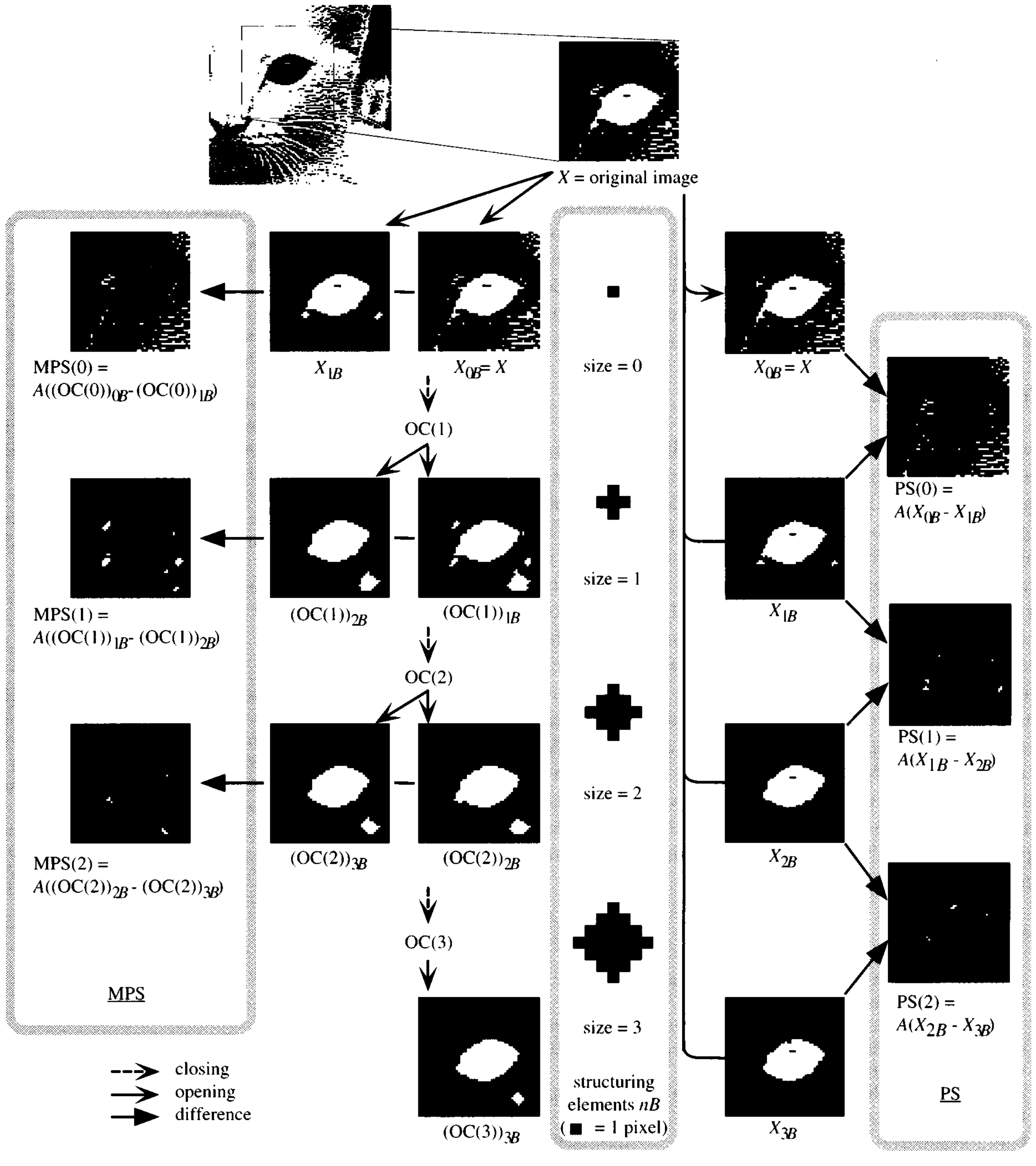


Fig. 4 Example of MPS and PS on a real image.

peak appears at size 6. Figure 3(b) shows the square with a spot of one pixel, and the spectra are shown in Table 1(b). In PS peaks appear only at sizes 2 and 3, and the peak at size 6 does not appear. On the other hand, MPS still extracts the peak for whole rectangle. Figure 3(c) shows the square with a spot of 3×3 pixels, and the spectra are shown in Table 1(c). In this case, the MPS extracts the narrow part above the spot in the square as well as the whole square, since the size of this part is as small as the size of the spot.

Secondly, we show an example on a real image while displaying intermediate images during the processes to calculate the conventional PS and the MPS. The process for the PS is shown on the right side in Fig. 4, and the MPS on the left side. In this figure OC(X, B, n), PS(X, B, n), MPS(X, B, n), are abbreviated to OC(n), PS(n), and MPS(n), respectively. Note that the actual spectral values of MPS and PS for each size are the areas of the white region in each image denoted as MPS() and PS(), respectively. The structuring element B and its magnified images are shown in the central column of Fig. 4.

In the process to calculate the PS, a small spot in the cat's eye is preserved for all n, and it is found in X_{3B} that the whole eye is about to be decomposed from around the spot. Thus, the influence of the spot is still found in PS(2). This influence will not disappear for larger sizes, and it is not possible to extract approximate shape of the whole eye. On the contrary, in the process to calculate the MPS, the influence of the hole does not appear in MPSs for the sizes larger than 1, because of the effect of closings. Thus the shape of the cat's eye itself can be extracted though the figure has a spot. It is also observed that the images $(OC(X, B, n))_{nB}$, $n = 0, 1, 2, \ldots$ form a sequence of "morphological multiresolution" images according to the extent of the SEs.

4. Concluding Remarks

Here we have shown the concept of MPS. MPS can extract approximate information of shape of figures in an image and distinguish it from the influence of small spots. If the spots are regarded to be caused by noise, the original image is separated from the effect of noise. It suggests that MPS is applicable to the design of nonlinear filters as the Fourier spectrum is applied to the linear ones. The design of nonlinear filters are very difficult problem and recently the applications of the learning methods have attracted much attention [7], [8]. However, the learning optimization method always requires an ideal image and it causes the problem whether the optimized filter can be applied for the other noisy images. MPS can achieve an unsupervised learning method using the criterion that the portion of small sizes are regarded as noises and should be eliminated, like "low-pass" filtering in the frequency analysis. We are now working on this.

References

[1] J. Serra, "Image Analysis and Mathematical Morphology," Academic Press, London, 1982.

[2] P. Maragos, "Pattern spectrum and multiscale shape representation," IEEE Trans. Pattern Anal. & Mach. Intell., vol. 11, pp. 701-716, 1989.

[3] A. Toet, "A hierarchical morphological image decomposition," Pattern Recognit. Lett., vol. 11, pp. 267-274, 1990.

[4] E. R. Dougherty, J. T. Newell, and J. B. Pelz, "Morphological texture-based maximum likelihood pixel classification based on local granulometric moments," Pattern Recognition, vol. 25, no. 10, pp. 1181-1198, 1992.

[5] E. R. Dougherty and Y. Cheng, "Morphological pattern-spectrum classification of noisy shapes: Exterior granulometries," Pattern Recognition, vol. 28, no. 1, pp. 81-98, 1995.

[6] P. Maragos and R. W. Schafer, "Morphological systems for multidimensional signal processing," Proc. IEEE, vol. 78, no. 4, pp. 690-710, 1990.

[7] A. Asano, K. Matsumura, K. Itoh, Y. Ichioka, and S.Yokozeki, "Optimization of morphological filters by learning," Opt. Commun., vol. 112, pp. 265-270, 1994.

[8] A. Asano, T. Yamashita, and S. Yokozeki, "Learning optimization of morphological filters with gray scale structuring elements," Opt. Eng., vol. 35, no. 8, pp. 2203-2213, 1996.

Akira Asano received the B.Eng., M.Eng., and Dr.Eng. degrees in Applied Physics from Osaka University, Osaka, Japan, in 1987, 1989, and 1992, respectively. In September and October in 1990, while he was a doctor candidate, he was a guest scientist in the Institute of Information Transmission Problems of the USSR Academy of Sciences, Moscow, USSR. Since 1992, he has been a research associate in the Department of Mechanical System Engineering, Kyushu Institute of Technology. From May 1994 to February 1995, he stayed as a guest researcher in VTT (Technical Research Centre of Finland) Information Technology, Tampere, Finland. His research interests include nonlinear image filtering, morphological image transformation, image restoration by blind deconvolution, and optical information processing. He is a secretary of the Intelligent Signal Processing Research Group of the Institute of Electronics, Information, and Communication Engineers, and a member of the steering committee of the Image Science Research Group of the Optical Society of Japan. He is also a member of Japan Society of Applied Physics and the Society of Photo-Optical Instrumentation Engineers. He received the award for excellent poster presentations in the 24th Joint Conference on Imaging Technology (Tokyo, Japan) in 1993.
E-mail: asano@mse.kyutech.ac.jp
WWW: http://joker.mse.kyutech.ac.jp/~asano/

Shunsuke Yokozeki is a professor of precision engineering at Kyushu Institute of Technology in Japan since 1990. He received the BS, MS, and Dr. of Engineering in 1962, 1967, and 1972, respectively, from Osaka University, Osaka, Japan. He was an assistant and an associate professor at the Osaka University during 1967-1987 and 1987-1990, respectively.